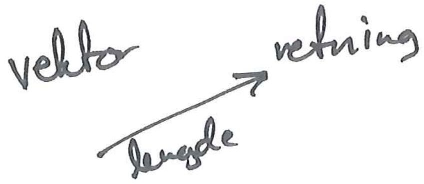
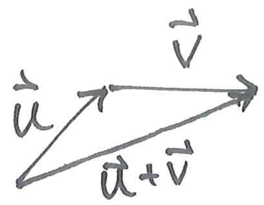
Øvinger onsdag

14:30 PI 248

16:30 PI 246




(1)

skalarmultiplikasjon

$\vec{v}$ $\quad t\cdot v$

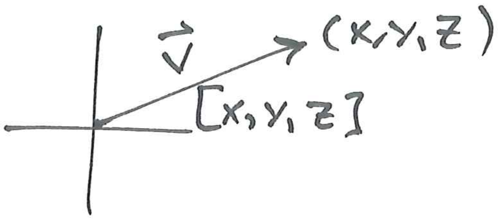
$t=-1$ $\quad \vec{v}$ $\quad -1\cdot\vec{v} = -\vec{v}$ motsattvektoren

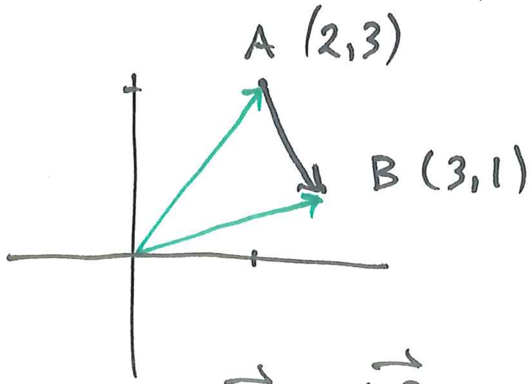
$$\vec{u}-\vec{v} = \vec{u} + (-\vec{v})$$

Koordinatsystem

Vektorer $\longleftrightarrow$ Punkt

$\vec{v} = \vec{OP}$ $\longleftrightarrow$ $P$




$\vec{OB} = [3,1]$

koordinatform

$\vec{OA} = [2,3]$

$$\vec{OA} + \vec{AB} = \vec{OB}$$

Så $\quad \vec{AB} = \vec{OB} - \vec{OA}$

$$= [3,1] - [2,3]$$

$$= [3-2,\ 1-3] = [1,-2]$$

(2) $A(x_1, y_1)$ $\qquad$ $B(x_2, y_2)$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [x_2, y_2] - [x_1, y_1]$$
$$= [x_2 - x_1, y_2 - y_1]$$

punkter!

$\cancel{(x_2, y_2) - (x_1, y_1)}$ ungå å skrive dette.

$A(2,3)$ $\qquad$ $B(3,1)$ $\qquad$ $C(-1,1)$ $\qquad$ $D(-2,4)$

$$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = [-2,4] - [-1,1] = [-1,3]$$
$$\overrightarrow{DC} = -\overrightarrow{CD} = -[-1,3] = [1,-3].$$

oppgave $\overrightarrow{CD} + 2\overrightarrow{DB} + 3\overrightarrow{BC} = 2(\overrightarrow{DB} + \overrightarrow{BC}) + \overrightarrow{BC} + \overrightarrow{CD}$

$$= 2\overrightarrow{DC} + \overrightarrow{BD} = \overrightarrow{DC} + \overrightarrow{BD} + \overrightarrow{DC} = \underline{\overrightarrow{DC} + \overrightarrow{BC}}$$

$$\overrightarrow{BC} - \overrightarrow{AC} = \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA} = -\overrightarrow{AB}$$
$$= -[1,-2] = [-1,2].$$

$$2\overrightarrow{AC} + \underbrace{\overrightarrow{CA}}_{-\overrightarrow{AC}} - \overrightarrow{DC} = 2\overrightarrow{AC} - \overrightarrow{AC} + \overrightarrow{CD}$$
$$= (2-1)\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$$
$$= \overrightarrow{OD} - \overrightarrow{OA} = [-2,4] - [2,3] = \underline{[-4,1]}$$

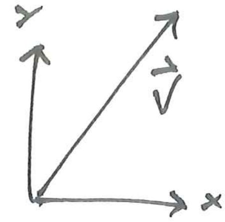
$$2\overrightarrow{AD} + \overrightarrow{CD} - 3DA + \overrightarrow{AC} = 3\overrightarrow{AD} - 3\overrightarrow{DA}$$
$$= 3\overrightarrow{AD} + 3\overrightarrow{AD} = 6\overrightarrow{AD}$$

summen er $\overrightarrow{AD}$

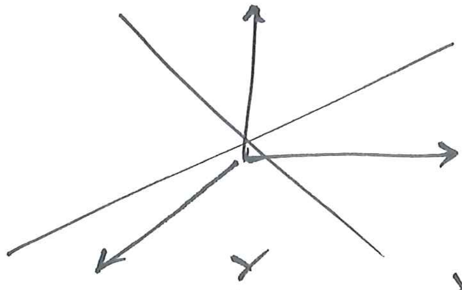
$$= 6[-4,1] = \underline{[-24, 6]}$$

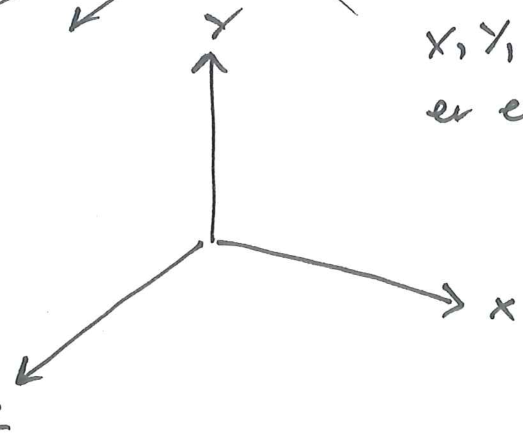
(3)

$\mathbb{R}^2$
planet




$\mathbb{R}^3$
rommet






x, y, z -aksene er et høyrehandssystem.

Vektorer

$\vec{v} = \overrightarrow{OP}$

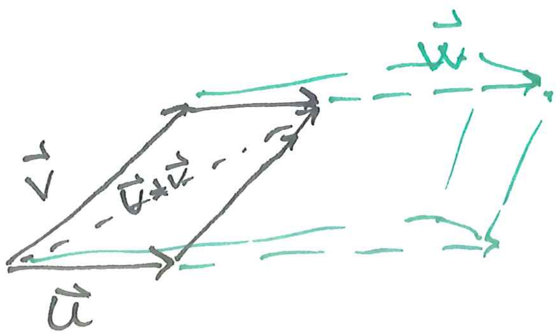
$[x, y, z]$

punkt

$P$

$(x, y, z)$

addisjon skalarmultiplikasjon
har samme egenskaper som i $\mathbb{R}^2$




parallellepipeder

$(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$

Basisvektorer i $\mathbb{R}^3$

(4)

$\vec{e}_1 = [1, 0, 0]$

$\vec{e}_2 = [0, 1, 0]$

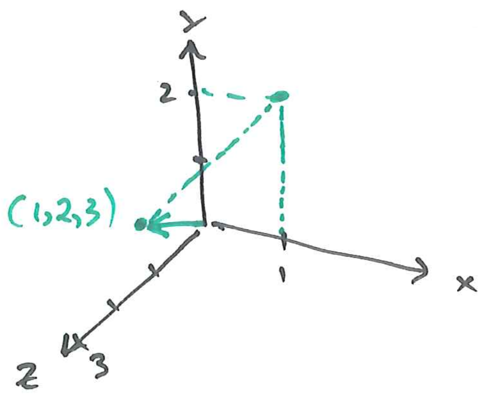
$\vec{e}_3 = [0, 0, 1]$

$$[x, y, z] = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$$

Eks

$$[1, 2, 7] + [4, -2, 3]$$
$$= [1+4, 2+(-2), 7+3] = [5, 0, 10]$$

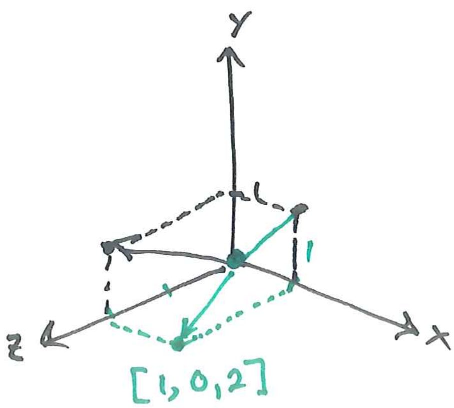
$$7[1, 2, 7] = [7\cdot 1, 7\cdot 2, 7\cdot 7]$$
$$= [7, 14, 49].$$




$\vec{v} = [1, 2, 3]$

Tegn opp $[1, 0, 2]$ $[0, 1, 2]$ $[1, 1, 3/2]$

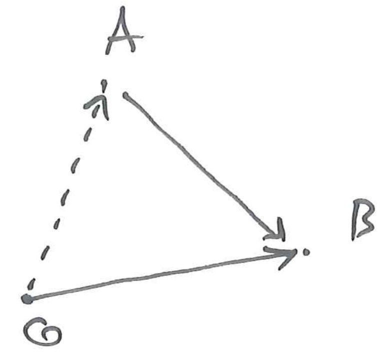
ser ut som $\vec{0}$.



La $B = (1, 2, -1)$

⑤ $\vec{AB} = [1, 0, 3]$

Finn koordinaten til A




$$\vec{OA} = \vec{OB} + \vec{BA} = \vec{OB} - \vec{AB}$$

$$= [1, 2, -1] - [1, 0, 3]$$

$$= [1-1, 2-0, -1-3]$$

$$\vec{OA} = [0, 2, -4]$$

Koordinaten til A er $\underline{(0, 2, -4)}$

Til orientering

$\mathbb{R}^n$

n-dimensjonalt Euklidsk rom

n-vektorer

$\vec{x} = [x_1, x_2, \ldots, x_n]$

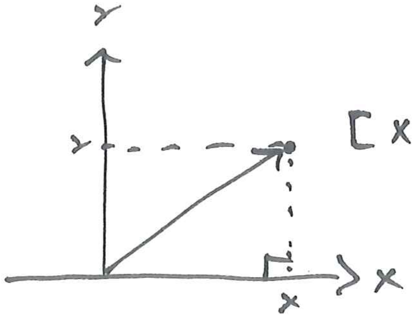
addisjon skalarmultiplikasjon

har egenskaper som i $\mathbb{R}^2$ og $\mathbb{R}^3$.

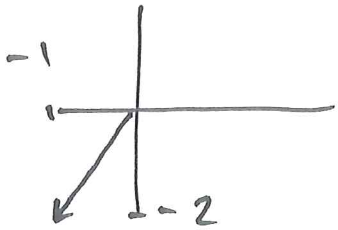
Det er vanskelig å visualisere $\mathbb{R}^n$ for $n \geq 4$.

# Norm, absoluttverdi, størrelse til vektorer

(6)




$[x, y] = \vec{v}$




lengden til $\vec{v}$ i koordinatsystemet er:

$$\|\vec{v}\| = |\vec{v}| = \sqrt{|x|^2 + |y|^2} = \sqrt{x^2 + y^2}$$

eks. $|[-1, -2]| = \sqrt{(-1)^2 + (-2)^2} = \underline{\sqrt{5}}$

$A(x_1, y_1)$ $\qquad$ $B(x_2, y_2)$

$$\vec{AB} = [x_2 - x_1, y_2 - y_1]$$

$$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

oppgave

$A = (5, -2)$ $\qquad$ $B = (10, 10)$

Hva er avstanden mellom A og B, lengden til vektoren $\vec{AB}$?

$$\vec{AB} = \vec{OB} - \vec{OA} = [10, 10] - [5, -2] = [5, 12]$$

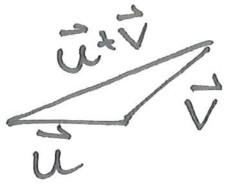
$$|\vec{AB}| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = \underline{\underline{13}}$$

Egenskaper til normen $|\vec{v}|$ til vektoren $\vec{v}$

7 * $|z \cdot \vec{v}| = |t|\,|\vec{v}|$

* $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

trekantulikheten



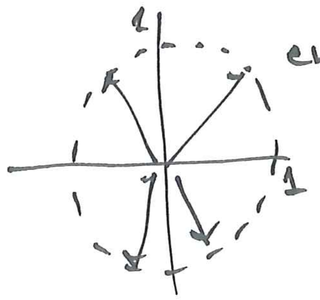

* $|\vec{v}| \geq 0$ og $|\vec{v}| = 0 \iff \vec{v} = \vec{0}$

$\vec{v} \neq \vec{0}$ da vil $|\vec{v}| \neq 0$

$$\left| \frac{1}{|\vec{v}|} \vec{v} \right| = \frac{1}{|\vec{v}|} |\vec{v}| = 1$$

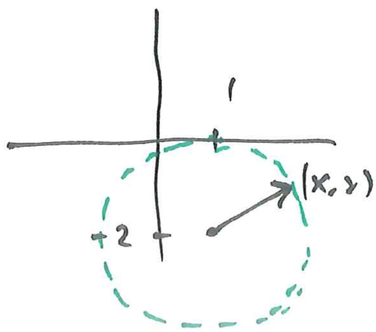
enhetsvektor (lengden er 1)

$$\vec{v} = \underbrace{|\vec{v}|}_{\text{størrelsen}} \cdot \underbrace{\frac{\vec{v}}{|\vec{v}|}}_{\text{retningen}}$$




enhetsvektoren i $\mathbb{R}^2$

Eksempel: Beskriv alle punkt $P(x,y)$ med avstand 2 til punktet $A(1,-2)$.




$|\overrightarrow{AP}| = 2$

$\overrightarrow{AP} = [x-1,\ y-(-2)] = [x-1,\ y+2]$

$|\overrightarrow{AP}|^2 = 2^2 = 4 = (x-1)^2 + (y+2)^2$

$4 = x^2 - 2x + 1 + y^2 + 4y + 4$

$\underline{x^2 - 2x + y^2 + 4y + 1 = 0}$

Hva er radius og senter til sirkelen gitt ved

(8) $$\underbrace{x^2+2x}_{(x+1)^2-1^2} + \underbrace{y^2-8y}_{(y-4)^2-(-4)^2} + 8 = 0$$

$$(x+1)^2 + (y-4)^2 \underbrace{-1-16+8}_{-9} = 0$$

$$(x+1)^2 + (y-4)^2 = 9$$

Løsningsmengden er en sirkel med senter i $\underline{(-1, 4)}$ og ~~Ren~~ radius $\underline{3}$