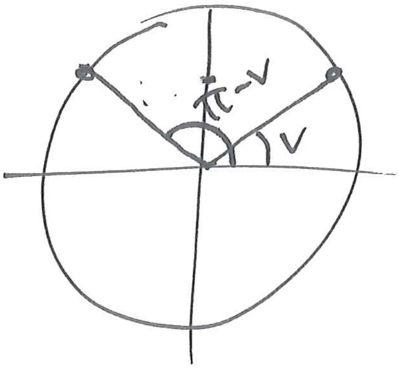
28 sep. 2018 Fausk

Løs likningen $5\sin(v) - 1 = 2$ (radian) $v \in [0, 2\pi)$

(1)




$$5\sin(v) = 2+1 = 3$$
$$\Updownarrow$$
$$\sin(v) = 3/5 = 0.6$$

$$v = \arcsin(0.6) = 0.6435 \text{ rad}$$
$(\sim 36.9°)$

$$u = \pi - v = 2.498 \text{ rad}$$

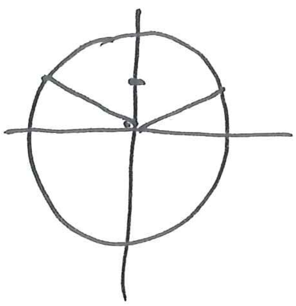
Løsningen er $\underline{0.6435 \text{ rad}}$ og $\underline{2.498 \text{ rad}}$

---

Løs likningen $\sin(2x+1) = 1/2$

$0 < x < 5$ (radianer)

La $2x+1 = v$ : $\sin(v) = 1/2$



$$v = \frac{\pi}{6} + 2\pi \cdot n$$
$$v = \frac{5\pi}{6} + 2\pi \cdot n$$
($n$ heltall)

(30° og 150° opp til hele omløp)

$2x+1 = v$ så $x = \frac{1}{2}(v-1)$

Løsningene er (uten krav til $x$)

$$x = \frac{\pi}{12} - \frac{1}{2} + \pi \cdot n$$
$$x = \frac{5\pi}{12} - \frac{1}{2} + \pi \cdot n$$

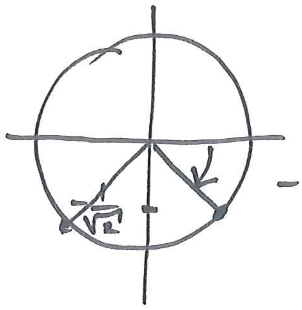
Løsningene mellom 0 og 5 er:

$$\underline{\{0.8089,\ 2.9033,\ 3{,}9505\}}$$

(2)

Løs likningen.

<u>oppgave</u> $\sin\left(2x + \frac{\pi}{3}\right) = \frac{-1}{\sqrt{2}}$ $\quad x \in [0, 2\pi)$




$V = 2x + \pi/3$

Løser først $\sin(V) = \frac{-1}{\sqrt{2}}$

$$V = -\frac{\pi}{4}(\text{rad}) + 2\pi \cdot n$$

$$V = \pi - \left(-\frac{\pi}{4}\right) + 2\pi \cdot n = \frac{5\pi}{4} + 2\pi \cdot n$$

Vi har $2x = V - \pi/3$

$$x = \frac{1}{2}(V) - (\pi/6) = \frac{1}{2}(V - \pi/3)$$

Så $x = \frac{1}{2}\left(-\frac{\pi}{4} - \frac{\pi}{3} + 2\pi \cdot n\right)$

$$= -\frac{7}{24}\pi + \pi \cdot n$$

og $x = \frac{1}{2}\left(\frac{5\pi}{4} - \frac{\pi}{3} + 2\pi \cdot n\right)$

$$= \frac{11\pi}{24} + \cancel{2}\pi \cdot n$$

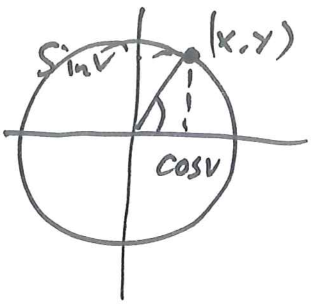
Løsningen i intervallet $[0, 2\pi)$ er

$-\frac{7}{24}\pi + \pi \cdot n \quad n = 1, 2 \quad : \quad \underline{\frac{17\pi}{24} \text{ og } \frac{17\pi}{24} + \pi}$

$\frac{11\pi}{24} + \pi \cdot n \quad n = 0, 1 \quad : \quad \underline{\frac{11\pi}{24} \text{ og } \frac{11\pi}{24} + \pi}$

Løs likningen

(3)




$$\tan v = 1$$

$$\frac{y}{x} = 1$$

$$v = \arctan(1) + \pi \cdot n$$

$$= \frac{\pi}{4} + \pi \cdot n \qquad n \text{ heltall.}$$

$\cos^2 x + \sin x = 5/4$

(4) Benytter Pytagoras teorem

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

Gjør om likningen til en 2.grads likning i $\sin x$ :

$$1 - \sin^2 x + \sin x = 5/4$$

$$\sin^2 x - \sin x + 1/4 = 0$$

$$\left(\sin x - \tfrac{1}{2}\right)^2 = 0$$

$$\Leftrightarrow \quad \sin x - \tfrac{1}{2} = 0 \Leftrightarrow \sin x = \tfrac{1}{2}$$

$$x = \arcsin\left(\tfrac{1}{2}\right) + 2\pi \cdot n$$

$$\left(\pi - \arcsin \tfrac{1}{2}\right) + 2\pi \cdot n$$

$$\underline{x = \frac{\pi}{6} + 2\pi \cdot n}$$

og $\underline{x = \frac{5\pi}{6} + 2\pi \cdot n}$

Oppgave

$$2\cos^2 V + \sqrt{3}\cos V - 3 = 0$$

$V \in [0, 2\pi)$

Løs likningen.

5

Dette er en 2.gradslikning i $\cos(V)$

Likningen er sann når $\cos(V) = \ldots$

eller $\cos(V) = \ldots$

$x = \cos V$ : $2x^2 + \sqrt{3}x - 3 = 0$

$$x = \frac{-\sqrt{3} \pm \sqrt{(\sqrt{3})^2 - 4\cdot 2\cdot(-3)}}{4}$$

$$= \frac{-\sqrt{3} \pm \sqrt{3 + 8\cdot 3}}{2\cdot 2} = \frac{-\sqrt{3} \pm \sqrt{3\cdot 9}}{2\cdot 2}$$

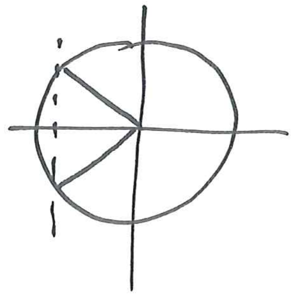
$$= \frac{-\sqrt{3} \pm \sqrt{3}\cdot 3}{4}$$

$$x = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \quad \text{og} \quad x = \frac{-4\sqrt{3}}{4} = -\sqrt{3}$$

$\cos(V) = -\sqrt{3} \sim -1.7$ ingen løsning

eller

$\cos(V) = -\sqrt{3}/2 = -0.866$



$$V = \frac{5\pi}{6} + 2\pi\cdot n$$

og $V = \frac{7\pi}{6} + 2\pi\cdot n$

I intervallet $[0, 2\pi)$

er løsningene $V = \frac{5\pi}{6}$ og $\frac{7\pi}{6}$

Gjennomgang av oppgave 7.3 i oblig 1.

$$\frac{1}{2x} + 2 \le \frac{2}{3}$$

6

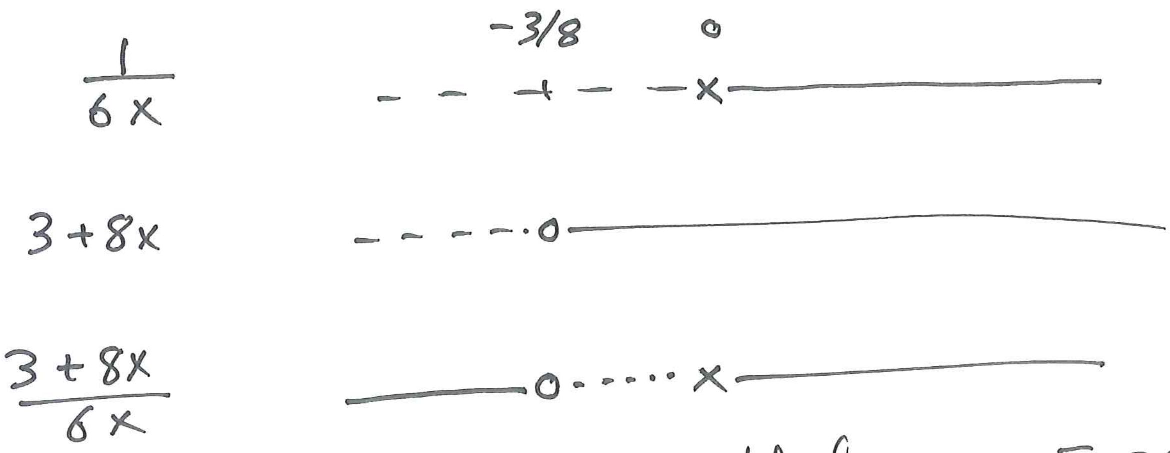
$$\frac{1}{2x} + 2 - \frac{2}{3} \le 0$$

$$\frac{1}{2x} + \frac{4}{3} \le 0$$

$$\frac{3}{6x} + \frac{8x}{6x} \le 0$$

$$(3+8x) \cdot \frac{1}{6x} \le 0$$

Fortegnskjema




Løsningen til ulikheten er $[-3/8, 0\rangle$

Alternativt gangert med $6x$

I $\quad 3 + 12x \leq 4x \qquad x > 0$

7

$8x \leq -3$ deler med 8

$x \leq -3/8 \qquad (\text{og } x > 0)$

ingen løsning.

II $\quad 3 + 12x \geq 4x \qquad x < 0$

$x \geq -3/8 \quad \text{og} \quad x < 0$

Løsningene er alle $x$ slik at

$-3/8 \leq x < 0$

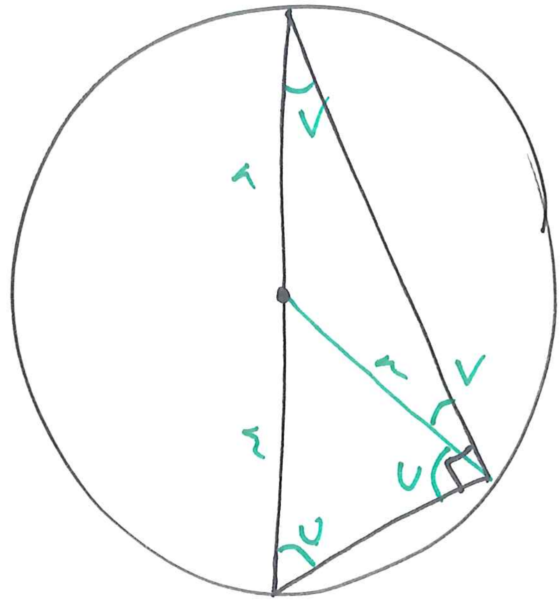
Kombinert gir dette løsningsmengden

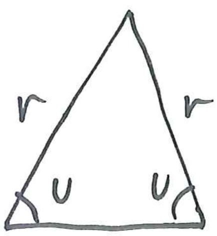
$$\underline{\underline{[-3/8, 0\rangle}}$$

(8)

## Thales teorem

Alle innskrevne trekanter i en sirkel hvor ene siden er en diagonal (går gjennom sentrum) er rettvinkla.




likebeinet

Summen av vinklen $u$, $(u+v)$ og $v$ må være $180°$ (vinklene i en trekant)

$$u+(u+v)+v=2(u+v)=180°$$

så $\underline{u+v=90°}$