11

I

a $f(x) = e^{-3x+1} - \ln(2x-1) + 2$ $\quad D: (\frac{1}{2}, \infty)$

$$f'(x) = -3 \cdot e^{-3x+1} - \frac{1}{2x-1} \cdot 2$$

b $f(x) = e^{-3x} + e^{3x} + 2$ $\quad D\ (-\infty, \infty)$

$$f'(x) = -3 \cdot e^{-3x} + 3 \cdot e^{3x}$$

c. $f(x) = \frac{1}{\ln(x^2)}$ $\quad D\ (-\infty, 0) \cup (0, \infty) \setminus \{-1, 1\} = \mathbb{R} \setminus \{-1, 0, 1\}$

$$f'(x) = \frac{-\frac{1}{x^2} \cdot 2x}{(\ln(x^2))^2} = -\frac{2}{x} \cdot \frac{1}{(\ln(x^2))^2}$$

Enklere: $f(x) = \frac{1}{2\ln|x|}$, $\quad f'(x) = \frac{1}{2x(\ln|x|)^2}$

d $f(x) = \ln(4 \cdot (x^3 - 2x + 1)^{1/5})$ $\quad D\ (-1.62, 0.62) \cup (1, \infty)$

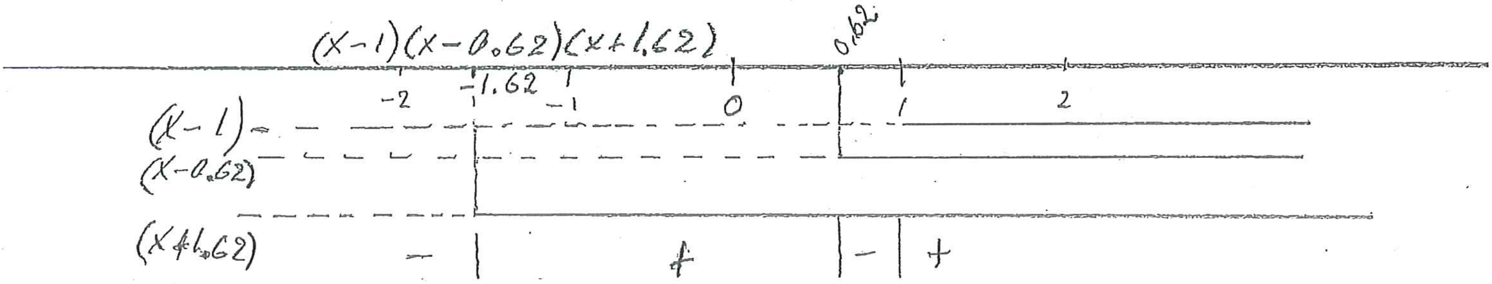
$$f'(x) = \frac{1}{4(x^3 - 2x + 1)^{1/5}} \cdot 4 \cdot \frac{1}{5}(x^3 - 2x + 1)^{-4/5} \cdot (3x^2 - 2)$$

$$= \frac{3x^2 - 2}{5(x^3 - 2x + 1)}$$

$x^3 - 2x + 1 : x - 1 = x^2 + x - 1$ $\quad$ Test $x = +1$

$$x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2} = \begin{cases} \frac{\sqrt{5}-1}{2} \approx 0.62 \\ -\frac{1}{2}(1+\sqrt{5}) \approx -1.62 \end{cases}$$

$(x-1)(x-0.62)(x+1.62)$



C2

e $f(x) = \dfrac{e^{2x}}{x+2}$, $D\ (-\infty, -2) \cup (-2, \infty)$

$$f'(x) = \frac{2 \cdot e^{x}(x+2) + e^{2x}}{(x+2)^2}$$

$x<0$

$|x| = -x$

f $f(x) = \ln(-x) \cdot \log_8 |x| = \dfrac{\ln(-x) \cdot \ln|x|}{\ln 8}$, $D\ (-\infty, 0)$

$x<0$: $f(x) = \dfrac{\ln(-x)^2}{\ln 8}$

$$f'(x) = \frac{1}{\ln 8} \cdot 2 \cdot \ln(-x) \cdot \frac{1}{+x} \cdot +1 = \frac{2 \cdot \ln(-x)}{\ln(8)}$$

$x>0$: $f(x) = \dfrac{\ln(-x) \cdot \ln(x)}{\ln(8)}$

$$f'(x) = \frac{1}{\ln 8}\left(\frac{1}{-x} \cdot -1 \cdot \ln(x) + \ln(-x) \frac{1}{x}\right)$$

$$= \frac{1}{\ln(8)}\left(\frac{\ln x}{x} + \frac{\ln(-x)}{x}\right)$$

Se vedlegg

II

a $f(x) = 3 \cdot e^{-x^{1/2}/2} - 1$

$$f'(x) = 3 \cdot e^{-\frac{x^{1/2}}{2}} \cdot -\frac{1}{2} x^{-1/2} \cdot \frac{1}{2} = -\frac{3}{4} \frac{e^{-\sqrt{x}/2}}{\sqrt{x}}$$

b $f(x) = e^{-3x} \cdot \cos(2x)$

$$f'(x) = -3 \cdot e^{-3x} \cdot \cos(2x) + e^{-3x} \cdot -\sin(2x) \cdot 2$$

c $f(x) = x^8 \cdot \ln|x| - \frac{x^8}{8}$

$x > 0$: $f(x) = x^8 \cdot \ln x - \frac{x^8}{8}$

$x < 0$: $f(x) = x^8 \cdot \ln(-x) - \frac{x^8}{8}$

$x > 0$: $f'(x) = 8 \cdot x^7 \cdot \ln x + x^8 \cdot \frac{1}{x} - x^7 = 8x^7 \cdot \ln x$

$x < 0$: $f'(x) = 8x^7 \cdot \ln(-x) + x^8 \cdot \frac{1}{-x} \cdot -1 - x^7 = 8x^7 \cdot \ln(-x)$

$$f'(x) = 8x^7 \cdot \ln|x|$$

d $f(x) = -(x^2 + 2x + 2) \cdot e^{-x}$

$$f'(x) = (x^2 + 2x + 2) \cdot e^{-x} - (2x + 2) \cdot e^{-x}$$

$$= x^2 \cdot e^{-x}$$

C4

II forts

e. $f(x) = \ln(\ln|x|)$ $x \neq (-1, 0, 1)$

$$f'(x) = \frac{1}{\ln|x|} \cdot \frac{1}{x}$$

f. $f(x) = x^x = e^{x \cdot \ln x}$

$$f'(x) = e^{x \cdot \ln x} \cdot \left(\ln x + x \cdot \frac{1}{x}\right) = e^{x \cdot \ln x}(\ln x + 1)$$

$$= x^x \cdot (\ln x + 1)$$

III

a. $13^x = 23$

$x \cdot \ln 13 = \ln 23$

$$x = \frac{\ln 23}{\ln 13} = 1.22244$$

b. $3 \cdot \log(x) = 4$, $\log x = \frac{4}{3}$

$$x = 10^{4/3} = \sqrt[3]{10^4} = 10 \cdot \sqrt[3]{10} = 21.5443$$

c. $\frac{3}{4^{x-2}} = \frac{2}{23} \Rightarrow \frac{2 \cdot 4^x}{4^2} = 3 \cdot 23 = 69$, $4^x = 8 \cdot 69$

$$x = \frac{\ln(8) + \ln(69)}{\ln(4)} = 4.55426$$

III fonts

d $\log_2(2x-1) = 3 + \log_2(x+1)$

$$\log_2(2x-1) - \log_2(x+1) = 3$$

$$\log_2\left(\frac{2x-1}{x+1}\right) = 3$$

$$\frac{2x-1}{x+1} = 8$$

$$2x-1 = 8x+8$$

$$6x = -9$$

$$x = -\frac{3}{2}$$

e $2\log_2|x+\frac{1}{2}| - \log_2|x| = 1$

$$\log_2|x+\tfrac{1}{2}|^2 - \log|x| = 1$$

$$\log_2\left(\frac{(x+\frac{1}{2})^2}{|x|}\right) = 1$$

$$\frac{(x+\frac{1}{2})^2}{|x|} = 2$$

$$x^2+x+\frac{1}{4} = 2|x| = \begin{matrix}2x \\ -2x\end{matrix}$$

$|x| = x \quad x>0$

$|x| = -x \quad x<0$

$x>0 \quad x^2-x+\frac{1}{4} = 0$

$$x = \frac{1 \pm \sqrt{1-4\cdot\frac{1}{4}}}{2} = \frac{1}{2}$$

$x<0 \quad x^2+3x+\frac{1}{4} = 0$

$$x = \frac{-3 \pm \sqrt{9-1}}{2} = \frac{-3 \pm 2\sqrt{2}}{2}$$

III forts

f.

$$\log|-5^{x+1}+25^x|=1$$

$$|-5^{x+1}+25^x|=10$$

$|-\ldots|=0$ når $x=1$

$|-\ldots|>0$ når $x>1$

$$\Rightarrow \quad 25^x-5^{x+1}=10$$

Se vedlegg

$$(5^x)^2-5\cdot5^x-10=0$$

$$5^x=\frac{5\pm\sqrt{25+40}}{2}=\frac{5\pm\sqrt{65}}{2}$$

$$x\cdot\ln5=\ln\frac{5\pm\sqrt{65}}{2}$$

$$x=\frac{\ln\left(\frac{5\pm\sqrt{65}}{2}\right)}{\ln5}\approx\underline{\underline{1.166}}$$

IV

a. $\int\left(4x^3-\frac{2}{x^3}\right)dx=\underline{x^4+\frac{1}{x^2}+C}$

$(\tan x)'=\frac{1}{\cos^2x}=1+\tan^2x$

b. $\int(\sin x-\cos x+\tan^2x)\,dx$

$=-\cos x-\sin x+\int\left(-1+\frac{1}{\cos^2x}\right)dx$

$\underline{=-\cos x-\sin x-x+\tan x+C}$

4 c) $\int \frac{6x+5}{3x-2}\,dx$

polynomdivisjon gir $\frac{6x+5}{3x-2} = \frac{2(3x-2)+9}{3x-2}$

$= 2 + \frac{9}{3x-2}$

$\int \frac{6x+5}{3x-2}\,dx = \int 2 + 9\cdot\frac{1}{3x-2}\,dx$

$= 2x + 9\int \frac{1}{3x-2}\,dx$

benytter linsubst

$u = 3x-2$

$du = 3dx$

$= 2x + 9\int \frac{1}{u}\,\frac{1}{3}du$

$= 2x + 9 \cdot \frac{1}{3}\,\frac{1}{3x-2} + c \quad = \underline{2x + \frac{3}{3x-2} + c}$

konstant

d) $\int e^{-\pi x-2} + e \, dx$

$\int e^{+u}\,\frac{du}{-\pi} + \int e\,dx$

(hvor $u = -\pi x - 2$
$du = -\pi dx$)

$= \frac{-1}{\pi}e^{u} + e\cdot x + c$

$= \underline{\frac{-1}{\pi}e^{-\pi x-2} + e\cdot x + c}$

e) $\int \sqrt[4]{2x+5}\,dx = \int (2x+5)^{1/4}dx$

La $u = 2x+5$ $du = 2dx$

Linear substitusjon

$\int u^{1/4}\,\frac{1}{2}du = \frac{1}{2}\cdot\frac{u^{5/4}}{5/4}+c = \frac{2}{5}u^{5/4}+c$

$= \underline{\frac{2}{5}(2x+5)^{5/4}+c}$

f) $\int 3x^2(x^3+5)^4\,dx$ $u = x^3+5$

substitusjon $du = 3x^2\,dx$

$\int u^4\,du$

$= \frac{u^5}{5}+c = \underline{\frac{(x^3+5)^5}{5}+c}$

V

a. I $\sin(x+y) = \sin x \cos y + \cos x \cdot \sin y$

II $\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$

I + II $\sin(x+y) + \sin(x-y) = 2 \cdot \sin x \cdot \cos y$

$$
\begin{array}{l}
x+y = A \\
- \; x-y = B \\
\hline
2x = A+B \Rightarrow x = \frac{A+B}{2}, \\
2y = A-B \Rightarrow y = \frac{A-B}{2}
\end{array}
$$

$\Rightarrow \sin(A) + \sin(B) = 2 \cdot \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

Vi har $A = ax \quad B = bx$

$\Rightarrow \sin(ax) + \sin(bx) = 2 \cdot \sin\left(\frac{a+b}{2} x\right) \cdot \cos\left(\frac{a-b}{2} \cdot x\right)$

b. Se vedlagte figur 5b bakerst

$a = \frac{2\pi}{20}, \quad b = \frac{2\pi}{22}$

c. $r = \frac{a+b}{2}, \quad s = \frac{a-b}{2}$

$2r = a+b$

$2s = a-b$

$2r + 2s = 2a \Rightarrow a = r+s$

$2r - 2s = 2b \Rightarrow b = r-s$

$$\sin(rx)\cdot\cos(sx) = \frac{1}{2}\left(\sin((r+s)x) + \sin((r-s)x)\right)$$

$$\int \sin(rx)\cdot\cos(sx)\,dx = \frac{1}{2}\int \sin((r+s)x)\,dx + \frac{1}{2}\int \sin((r-s)x)\,dx$$

$$= \frac{1}{2}\cdot\frac{-1}{r+s}\cos((r+s)x) + \frac{1}{2}\cdot\frac{-1}{r-s}\cos((r-s)x)$$

$$= -\frac{1}{2(r+s)}\cos((r+s)x) - \frac{1}{2(r-s)}\cdot\cos((r-s)x)$$

VI

a $\quad x'(t) = -k\cdot x(t)$

$$x(t) = x_0\cdot e^{-kt}, \quad x'(t) = -k\cdot x_0\cdot e^{-kt}$$

Setter inn: $-kx_0\cdot e^{-kt} = -kx_0\cdot e^{-kt} \Rightarrow x(t)$ er en løsning

$$x(0) = x_0$$

b

$$x(t_{1/2}) = \frac{1}{2}x_0 = x_0\cdot e^{-kt_{1/2}}$$

$$\Rightarrow e^{k\cdot t_{1/2}} = 2$$

$$k\cdot t_{1/2} = \ln(2)$$

$$t_{1/2} = \frac{\ln(2)}{k}$$

VI forts

c

$$5700 \cdot k = \ln(2)$$

$$k = \frac{\ln(2)}{5700} = \underline{0.000121605 \text{ år}^{-1}}$$

d.

$$\frac{X(t)}{X(0)} = 0.729 = e^{-kt}$$

$$e^{kt} = \frac{1}{0.729}$$

$$k \cdot t = \ln \frac{1}{0.729}$$

$$\underline{t = 2599 \text{ år}}$$

e

$$2000 - 1200 = 800 \Rightarrow e^{-k \cdot 800} = \underline{0.907}$$

$$2000 - 1300 = 700 \Rightarrow e^{-k \cdot 700} = \underline{0.918}$$

VII

$$f(x) = x^r \cdot e^{-x}$$

a.

$$f'(x) = r \cdot x^{r-1} \cdot e^{-x} - x^r \cdot e^{-x}$$

$$f'(x) = 0 \Rightarrow e^{-x} \cdot x^{r-1}(r - x) = 0 \Rightarrow \underline{x = r}$$

$$f''(x) = r(r-1)x^{r-2} \cdot e^{-x} - r \cdot x^{r-1} \cdot e^{-x} - r \cdot x^{r-1} \cdot e^{-x} + x^r \cdot e^{-x}$$

$$f''(x) = e^{-x} \cdot x^{r-2} \cdot (r^2 - r - 2rx + x^2)$$

$$f''(r) = e^{-r} \cdot r^{r-2} \cdot (r^2 - r - 2r^2 + r^2)$$

$$f''(r) = -e^{-r} \cdot r^{r-1} < 0$$

$f'(x) = 0$ for $x = r \Rightarrow$ Vi har et ekstremalpunkt.

$f''(r)$ er negativ $\Rightarrow$ Vi har et maksimum.

b. $f'(x)$ er negativ når $x > r$, positiv når $x < r$.

$\Rightarrow f(x) = x^r \cdot e^{-x} \to 0$ når $x \to \infty$ Nei!

$f(r \cdot k) = \frac{(rk)^r}{e^{rk}} = \left(\frac{rk}{e^k}\right)^r$ siden $\frac{k}{e^k} \to 0$ når $k \to \infty$ følger det at $f(rk) \to 0$ når $k \to \infty$

$0 < \frac{k}{e^k} < \frac{k}{2^k}$ som åpenbart går mot null.

c. $g(x) = \ln x \cdot x^{-\frac{1}{n}}$

$$g'(x) = \frac{1}{x} \cdot x^{-\frac{1}{n}} + \ln x \cdot -\frac{1}{n} \cdot x^{-\frac{1}{n}-1}$$

$$= x^{-\frac{1}{n}-1} \cdot \left(1 - \ln x \cdot \frac{1}{n}\right)$$

$g'(x) = 0 \Rightarrow \underline{\ln x = n} \Rightarrow \underline{x = e^n}$

$g'(x) > 0: \ln x < n \Rightarrow \underline{x < e^n}$

$g'(x) < 0: \ln x > n \Rightarrow \underline{x > e^n}$

$\Rightarrow x = e^n$ er et maksimumspunkt.

$$g(e^n) = \ln(e^n) \cdot (e^n)^{-\frac{1}{n}} = n \cdot e^{-1} = \underline{\underline{\frac{n}{e}}}$$

Dette gir at $\lim_{x \to \infty} \frac{\ln x}{\sqrt[n]{x}} = 0$

Sett inn $u = \ln x$ i $\frac{u^r}{e^u}$ gir $\frac{(\ln x)^r}{x}$

tar vi n-te roten får vi $\frac{\ln x}{\sqrt[n]{x}}$.

$$\lim_{x \to \infty} \frac{x^r}{e^x} = \lim_{u \to \infty} \left(\frac{\ln u}{u^{1/n}}\right)^n = \left(\lim_{u \to \infty} \frac{\ln u}{u^{1/n}}\right)^n$$

VII forts

d $f''(x) = 0$

$\Rightarrow x^2 - 2rx + r^2 - r = 0$

$$x = \frac{2r \pm \sqrt{4r^2 - 4(r^2 - r)}}{2} = \frac{2r \pm 2\sqrt{r}}{2}$$

$\underline{x = r \pm \sqrt{r}}$ <u>Vendepkt til $f(x)$</u>

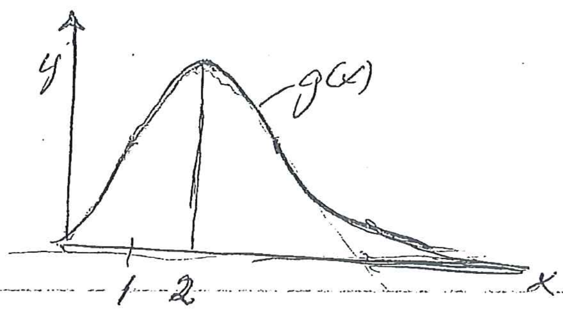
VIII

$g(x) = x^2 e^{-x}$

$g'(x) = 2x \cdot e^{-x} - x^2 \cdot e^{-x} = x \cdot e^{-x}(2-x)$

$g''(x) = 2 \cdot e^{-x} - 2x \cdot e^{-x} - 2x \cdot e^{-x} + x^2 e^{-x}$

$g''(x) = e^{-x}(2 - 4x + x^2)$



maxpkt. $g'(x) = 0 \Rightarrow x = 2$

$g'(x) > 0 \quad x < 2$

$g'(x) < 0 \quad x > 2$

Vendepkt. $g''(x) = 0 \Rightarrow x^2 - 4x + 2 = 0 \Rightarrow x = 2 \pm \sqrt{2}$

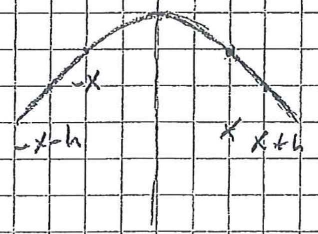
$g''(x) < 0 \quad 2 - \sqrt{2} < x < 2 + \sqrt{2}$

$g''(x) > 0 \quad 0 \le x < 2 - \sqrt{2} \cup x > 2 + \sqrt{2}$

Asymptote <u>$y = 0$.</u>

IX

$f(x)$ Symmetrisk

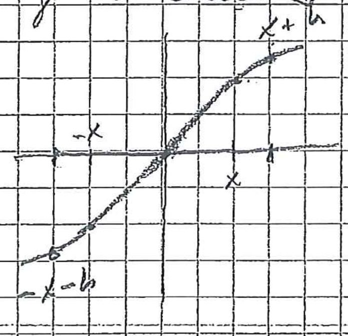


$$\frac{f(-x)-f(-x-h)}{h} \qquad \frac{f(x+h)-f(x)}{h}$$

$$f(x) = f(-x)$$

$$\frac{f(x)-f(x+h)}{h} = -\frac{f(x+h)-f(x)}{h} \quad \text{A. S.}$$

$f(x)$ antisymmetrisk



$$\frac{f(-x)-f(-x-h)}{h} \qquad \frac{f(x+h)-f(x)}{h}$$

$$f(x) = -f(-x) \text{ eller } f(-x) = -f(x)$$

$$\frac{-f(x)+f(x+h)}{h} = \frac{f(x+h)-f(x)}{h} \quad \text{S.}$$

$f(x)$ jevn : $f(x) = f(-x)$

Derivere $f(-x)$: $\frac{d}{dx}\left(f(-x)\right) = f'(-x)\cdot(-x)' = -f'(-x)$

$\frac{d}{dx}\left(f(x)\right) = f'(x)$

Derfor er $f'(x) = -f'(-x)$ og $f'(x)$ er en odde funksjon. $f(x)$ odde gir tilsvarende $f'(x)$ jevn

X

a Perioden blir større når man innfører friksjon.

$$x''(t) + \frac{c}{m} \cdot x'(t) + \frac{k}{m} \cdot x(t) = 0$$ Diff. lign.

$$x(t) = A \cdot e^{-Bt} \cdot \sin(D \cdot t)$$

Setter $x(t)$ inn i diff. ligning.

$$x'(t) = -AB \cdot e^{-Bt} \cdot \sin(Dt) + A \cdot D \cdot e^{-Bt} \cdot \cos(Dt)$$

$$x''(t) = AB^2 \cdot e^{-Bt} \sin(Dt) - ABD \cdot e^{-Bt} \cdot \cos(Dt)$$
$$- ABD \cdot e^{-Bt} \cdot \cos(Dt) - AD^2 \cdot e^{-Bt} \cdot \sin(Dt)$$

Dette settes inn i diff. ligning.
Faktoren $A \cdot e^{-Bt}$ finnes i alle ledd og kan settes utenfor en parentes

$$A \cdot e^{-Bt} \cdot \Big[B^2 \cdot \sin(Dt) - BD \cdot \cos(Dt) - BD \cdot \cos(Dt) - D^2 \cdot \sin(Dt)$$
$$- \frac{c}{m} \cdot B \cdot \sin(Dt) + \frac{c}{m} \cdot D \cdot \cos(Dt)$$
$$+ \frac{k}{m} \cdot \sin(Dt) \Big] = 0$$

./.

$$A \cdot e^{-Bt} \cdot \Big[\sin(Dt) \cdot \left(B^2 - D^2 - \frac{c}{m} \cdot B + \frac{k}{m}\right)$$

$$+ \cos(Dt) \cdot \left(-2BD + \frac{c}{m} D\right)\Big] = 0$$

For at dette skal bli null må faktorene

etter sin(Dt) og cos(Dt) være null.

$$\cos(Dt): \quad -2B\cancel{D} + \frac{c}{m} \cdot \cancel{D} = 0 \Rightarrow \underline{B = \frac{c}{2m}}$$

$$\sin(Dt): \quad \frac{c^2}{4m^2} - D^2 - \frac{c}{m} \cdot \frac{c}{2m} + \frac{k}{m} = 0$$

$$D^2 = \frac{k}{m} - \frac{c^2}{4m^2} = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$$

$$\underline{D = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}} = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}}$$

$$c = 0 \text{ gir } D = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$$

Hvis man innfører demping blir uttrykke

under kvadratroten mindre og T blir større.

b $\quad x'(0) = A \cdot D$

$x(t) = 0$ når $\sin(Dt) = 0 \Rightarrow Dt = 0, \pi, 2\pi, 3\pi$

Da er $\cos(Dt) = 1 \Rightarrow Dt = 1, -1, 1, -1$

og $t = \{0, \frac{\pi}{D}, \frac{2\pi}{D}, \frac{3\pi}{D}\}$

$$x'(t) = A \cdot D \cdot e^{-\frac{B \cdot \pi}{D}}, \; AD \cdot e^{-\frac{2B\pi}{D}}, \; AD \cdot e^{-\frac{3B\pi}{D}}$$

c. Da blir det ingen svingning.

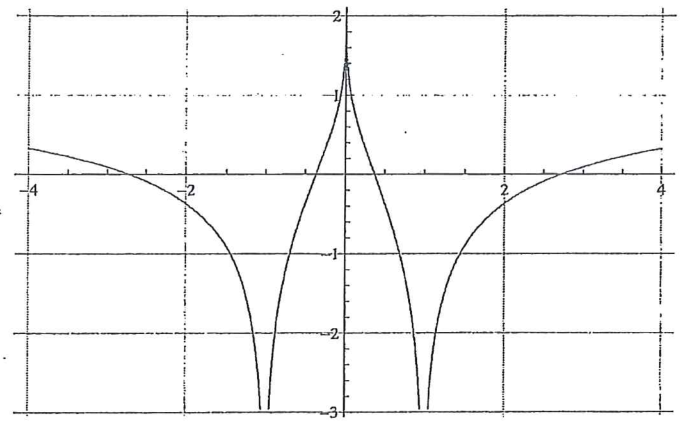
2e

In[1]:= h[x_] = Log[Abs[Log[Abs[x]]]]

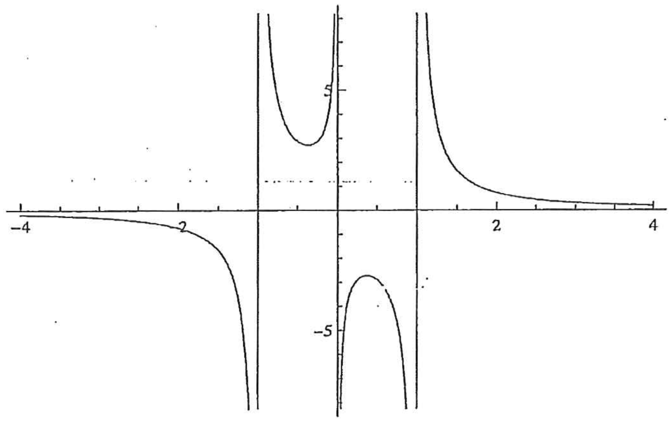
Out[1]= Log[Abs[Log[Abs[x]]]]

In[2]:= Plot[h[x], {x, -4, 4}, GridLines → Automatic]

Out[2]=



$$\text{Plot}\left[\frac{1}{\text{Log}[\text{Abs}[x]]\ x}, \{x, -4, 4\}\right]$$



56

In[5]:=

$$\text{Manipulate}\left[\text{Plot}\left[\text{Sin}[a\,x] + \text{Sin}\left[\frac{2\,\pi}{T}\,x\right], \{x, 0, 600\}, \text{GridLines} \to \text{Automatic}, \text{PlotPoints} \to \{200\}\right], \{T, 15, 25, 1\}\right]$$

Out[5]=