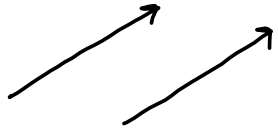
Vektorer

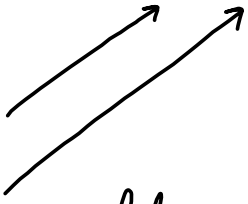
Vektor har lengde og retning

$\vec{v}$

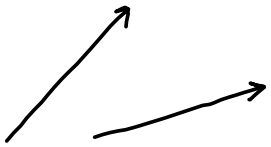
Størrelsen, absoluttverdien skrives $|\vec{v}|$. (ikke nødvendigvis gitt ved fysisk lengde)



like vektorer

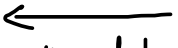


ulike vektorer



ulike vektorer

To vektorer er like hvis de har samme retning og størrelse.

$\vec{u}$  snur retningen  $-\vec{u}$

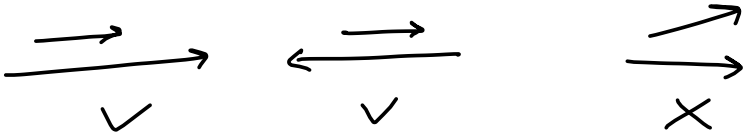
motsatt vektoren til $\vec{u}$.

Nullvektoren $\vec{0}$ har lengde 0. $\vec{0}$

$\vec{0}$ har alle (ingen!) retninger.

Hvis en vektor har lengde 0 så er den nullvektoren.

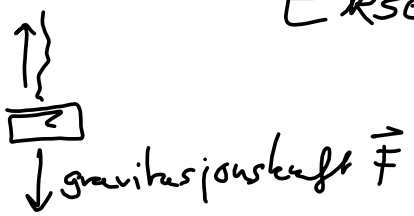
To vektorer er parallelle hvis de har samme retning eller motsatt retning

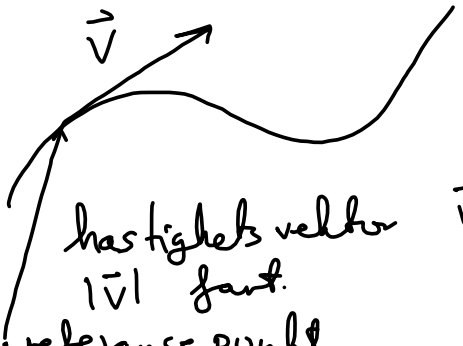


✓ ✓ ✗ (ikke parallell)

Alle vektorer er parallelle med nullvektoren.

Eksempler



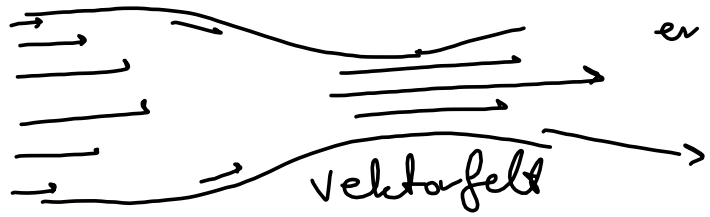


hastighets vektor $\vec{v}$

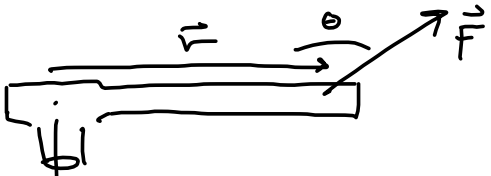
$|\vec{v}|$ fart.

referanse punkt.

posisjon fra et referansepunkt er en vektor $\vec{s}$.



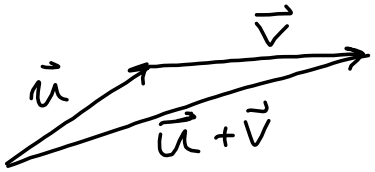
Kraftmoment



størrelse $|\vec{F}| \cdot |\vec{r}| \sin\theta$

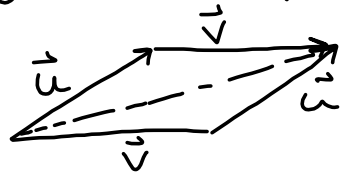
retning.

Sum av vektorer



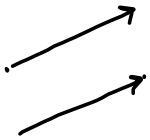
$\vec{u} + \vec{v}$

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$ addisjon av vektorer er kommutativ

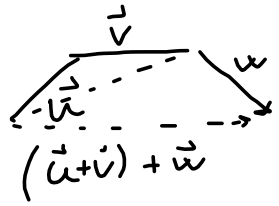


$\vec{0} + \vec{u} = \vec{u}$

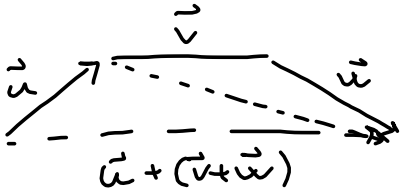
$\vec{u} + \vec{0} = \vec{u}$



$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ addisjon av vektorer er assosiativ

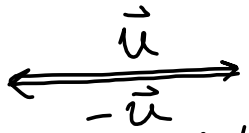


$(\vec{u}+\vec{v}) + \vec{w}$



$\vec{u} + (\vec{v}+\vec{w})$

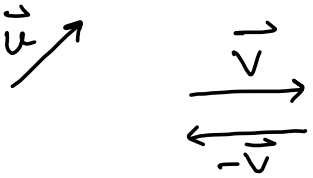
$\vec{u} + (-\vec{u}) = \vec{0}$



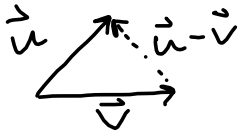
Motsatt vektoren er additivt inverselement

Differense av vektorer

$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$



$(\vec{u} - \vec{v}) + \vec{v} = \vec{u} + \underbrace{(-\vec{v} + \vec{v})}_{\vec{0}} = \vec{u}$



$\vec{v}$

$\vec{v} + \vec{v} + \vec{v} = 3\vec{v}$ $\quad \overrightarrow{3\vec{v}}$

$-\vec{v} + (-\vec{v}) =$
$2(-\vec{v}) = -2\vec{v}$ $\quad \overleftarrow{-2\vec{v}}$

$\frac{1}{2}\vec{v}$ $\quad \vec{v}$, $\overrightarrow{\frac{1}{2}\vec{v}}$

Skalarmultiplikasjon t reelt tall

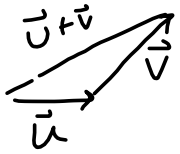
$$t\vec{v} = \begin{cases} \text{samme retning som } \vec{v}, \text{ lengde } t|\vec{v}| & t>0 \\ \vec{0} & t=0 \\ \text{motsatt retning av } \vec{v}, \text{ lengde } (-t)|\vec{v}| & t<0 \end{cases}$$

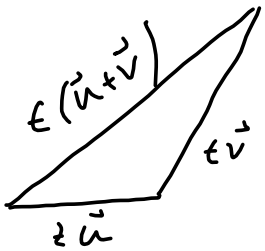
$\vec{v}$ $\quad \sqrt{2}\vec{v}$ $\quad -\frac{1}{3}\vec{v}$ er $\quad 0\cdot\vec{v} = \vec{0}$.

$t(s\vec{v}) = (ts)\cdot\vec{v}$

$s\vec{v} + t\vec{v} = (s+t)\cdot\vec{v}$

$t(\vec{v}+\vec{u}) = t\vec{v} + t\vec{u}$

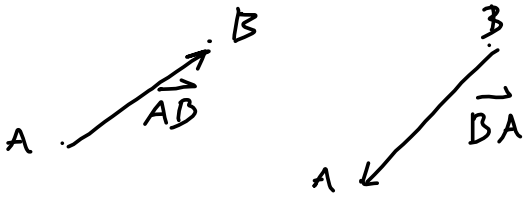




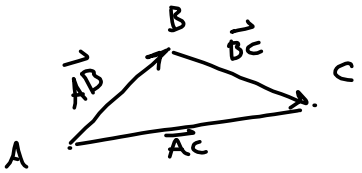
$-\vec{v} = (-1)\vec{v}$ fordi $(-1)\cdot\vec{v}$ har lengde $1\cdot|\vec{v}| = \vec{v}$ og motsatt retning til $\vec{v}$.

motsatt-vektoren

Vektorer definert fra punkt



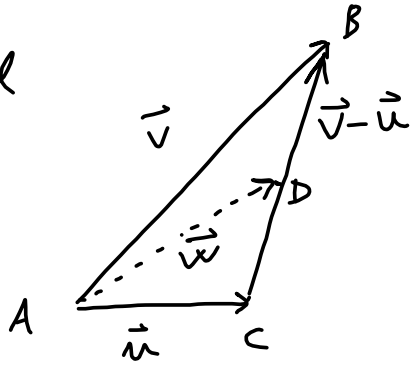
$-\vec{AB} = \vec{BA}$



$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{AC} - \vec{AB} = \vec{BC}$

Eksempel



$\vec{u} = \vec{AC}$, $\vec{v} = \vec{AB}$

$\vec{CB} = \vec{v} - \vec{u} \; (= \vec{CB})$

Hva er $\vec{w}$ uttrykt med $\vec{u}$ og $\vec{v}$.

$\vec{w} = \vec{AD} =$

$= \vec{u} + \frac{1}{2}(\vec{v} - \vec{u})$

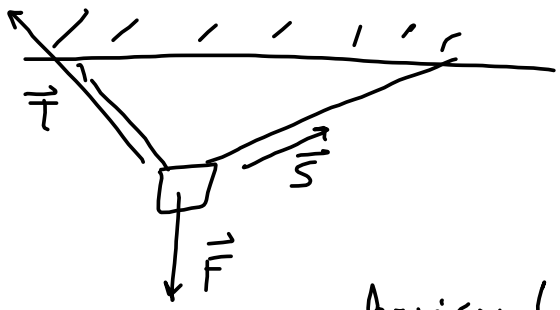
$= \vec{u} + \frac{1}{2}\vec{v} - \frac{1}{2}\vec{u}$

$= (1 - \frac{1}{2})\vec{u} + \frac{1}{2}\vec{v}$

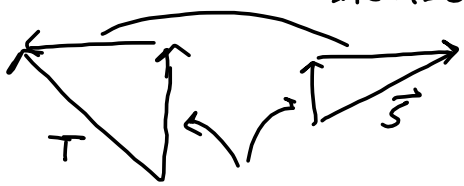
$\vec{w} = \frac{1}{2}\vec{u} + \frac{1}{2}\vec{v}$

$= \underline{\frac{1}{2}(\vec{u} + \vec{v})}$

Dekomponere vektorer



Horisontal komponentene like store og med motsatt retning



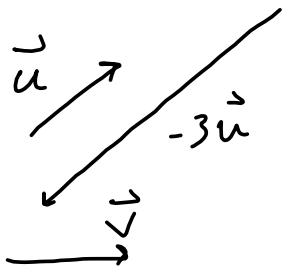
Summen av dei vertikale komponentene må vere like stor som $\vec{F}$ og med motsatt retning.

Forenkle

$\frac{1}{2}(3\vec{u} - 5\vec{v}) + 2\vec{v}$

$\frac{3}{2}\vec{u} - \frac{5}{2}\vec{v} + 2\vec{v} =$

$\frac{3}{2}\vec{u} + (-\frac{5}{2} + 2)\vec{v} = \underline{\frac{3}{2}\vec{u} - \frac{1}{2}\vec{v}}$



Ekv $\vec{u}$ og $\vec{0}$ parallelle? ja

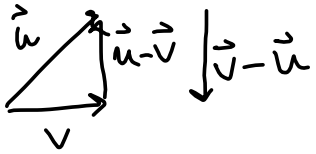
Ekv $2\vec{u}$ og $\vec{u}$ parallelle? ja

Ekv $-3\vec{u}$ og $\vec{u}$ parallelle? ja

Ekv $\vec{u} - \vec{v}$ og $\vec{v} - \vec{u}$ parallelle?

ja, dei er motsatt vektorer av hverandre

$-(\vec{u} - \vec{v}) = -\vec{u} + (-1)^2\vec{v} = \vec{v} - \vec{u}$



Ekv $\vec{u} - \vec{v}$ og $\vec{u} + \vec{v}$ parallelle? Nei.