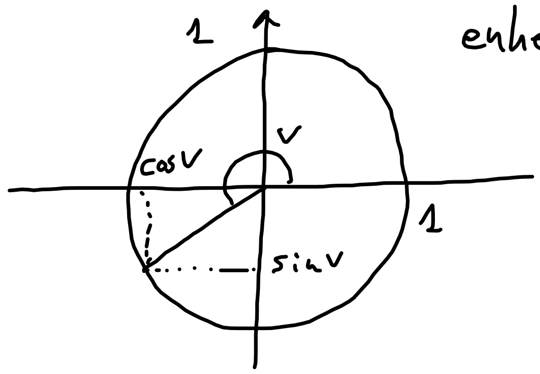


enhetssirkel

Pytagoras:

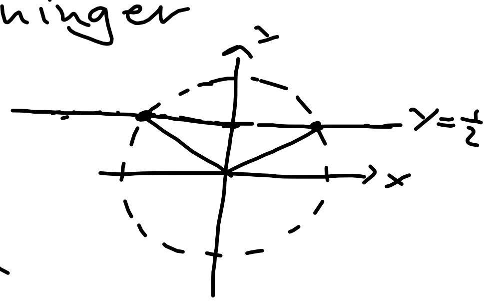
$\cos^2(v) + \sin^2(v) = 1$

for alle v.

(10.9 i boka)

Trigonometriske likninger

Eks $\sin(x) = \frac{1}{2}$



$x = 30° + 360° \cdot n$

n heltall

$x = 150° + 360° \cdot n$

I radianer: $x = \frac{\pi}{6} + 2\pi \cdot n$

Løsningene er $x = \frac{5\pi}{6} + 2\pi \cdot n$ n heltall

Løsningene til $\sin x = \frac{1}{2}$ $x \in [0, 4\pi]$

er $x = \frac{\pi}{6}, \frac{\pi}{6} + 2\pi, \frac{5\pi}{6}, \frac{5\pi}{6} + 2\pi$

$\sin(2x + 1) = \frac{1}{2}$

Først: løse $\sin(u) = \frac{1}{2}$

deretter: løse $2x + 1 = u$.

$2x+1 = u = \frac{\pi}{6} + 2\pi \cdot n$ $\quad$ $2x+1=u = \frac{5\pi}{6} + 2\pi \cdot n$

tar $+1$ over på høyre side og deler med 2:

Løsningene er:

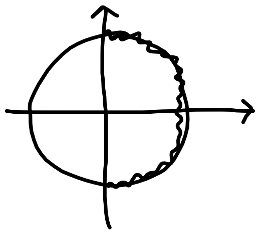
$$x = \frac{1}{2}\left(\frac{\pi}{6} - 1 + 2\pi \cdot n\right) = \underline{\frac{\pi}{12} - \frac{1}{2} + \pi \cdot n}$$

$$x = \frac{1}{2}\left(\frac{5\pi}{6} - 1 + 2\pi \cdot n\right) = \underline{\frac{5\pi}{12} - \frac{1}{2} + \pi \cdot n}$$

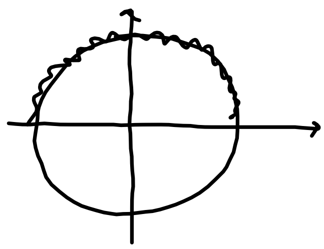
Utvider inversfunksjonene til sin, cos, tan

arcsin er inversfunksjonen til $\sin(x)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

For $y \in [-1, 1]$ så er $\arcsin(y)$ vinkelen mellom $-\frac{\pi}{2}$ og $\frac{\pi}{2}$ slik $\sin(\arcsin y) = y$



arccos er inversfunksjonen til $\cos(x)$, $x \in [0, \pi]$

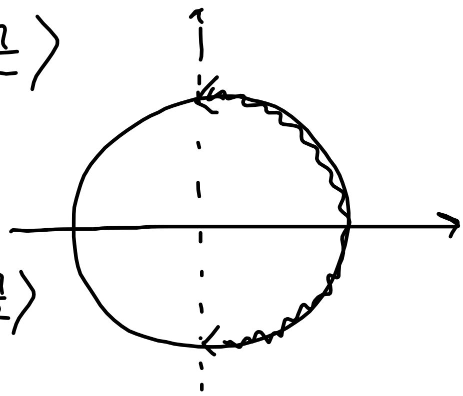


arctan er inversfunksjonen til $\tan(x)$, $x \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$

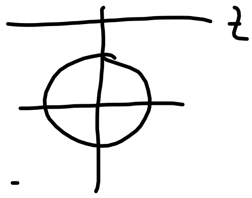
arctan er definert for alle reelle tall $\mathbb{R}$.

$\arctan(z)$ er vinkelen i $\left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$

$\tan(\arctan(z)) = z$

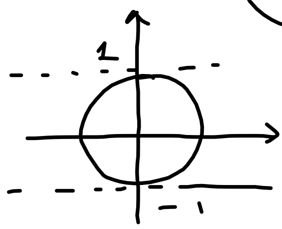


10.3 $\sin x = t$



$|t| > 1$ tom løsningsmengde

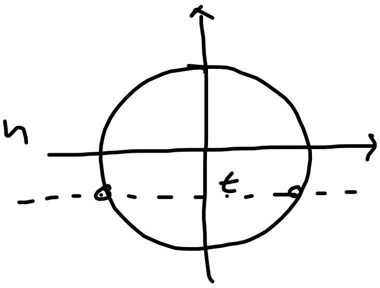
$t = 1$ $x = \frac{\pi}{2} + 2\pi \cdot n$



$t = -1$ $x = \frac{-\pi}{2} + 2\pi \cdot n$

$|t| < 1$ $x = \arcsin t + 2\pi \cdot n$

$x = \pi - \arcsin t + 2\pi \cdot n$



Eks $\sin x = \frac{-2}{3}$ $(\sim -42°)$

$x = \arcsin\left(-\frac{2}{3}\right) + 2\pi \cdot n \sim -0{,}729 + 2\pi \cdot n$

$x = \pi - \arcsin\left(-\frac{2}{3}\right) + 2\pi \cdot n \sim 3.87 + 2\pi \cdot n$

$3 \sin x + 2 = 0$

$3 \sin x = -2$

$\sin x = -2/3$ likningen ovenfor

$6 \sin^2 x - \sin x - 1 = 0$

2. grads likning i $\sin x$

$\sin^2 x - \frac{1}{6} \sin x - \frac{1}{6} = 0$

$\left(\sin x - \frac{1}{2}\right)\left(\sin x + \frac{1}{3}\right) = 0$

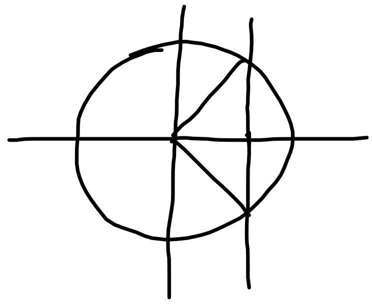
Løsningene til $\sin x$ er:

$\sin x = 1/2$ løs likningene ...

$\sin x = -1/3$

10.4

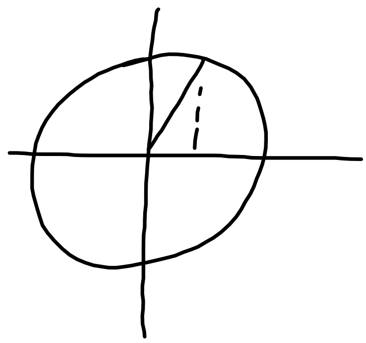
$$\cos x = \frac{1}{\sqrt{2}}$$



$$x = \arccos\left(\frac{1}{\sqrt{2}}\right) + 2\pi \cdot n$$
$$= \underline{\frac{\pi}{4} + 2\pi \cdot n}$$

$$x = -\arccos\left(\frac{1}{\sqrt{2}}\right) + 2\pi \cdot n$$
$$= \underline{-\frac{\pi}{4} + 2\pi \cdot n}$$

10.5 $\tan(x) = \sqrt{3}$



$\left(\tan(x+\pi) = \tan(x)\right)$

$$x = \underline{\frac{\pi}{3} + \pi \cdot n}$$

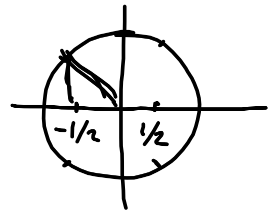
$$\sin^2 x - 3\cos^2 x = 0$$

1. metode Pyt: $\sin^2 = 1 - \cos^2$ setter inn

$$1 - \cos^2 x - 3\cos^2 x = 1 - 4\cos^2 x = 0$$
$$\cos^2 x = \frac{1}{4}$$

$(\cos x)^2 = \frac{1}{4}$ så $\cos x = -\frac{1}{2}$ og $\frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{-\pi}{3}, \frac{-2\pi}{3}$ opp til et helt omløp.



metode $\sin^2 x = 3\cos^2 x$

$\tan^2 x = 3 \; \ldots$ $(\cos x \neq 0)$