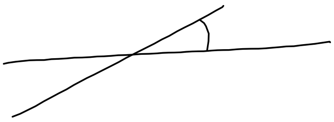
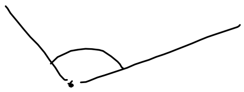
10.1 Naturlig vinkelmål, radianer



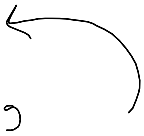
vinkel mellom linjer: minste vinkel den er mellom 0° og 90°.

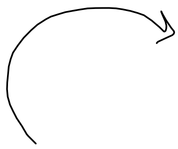


vinkel mellom to stråler (minste vinkel) mellom 0° og 180°

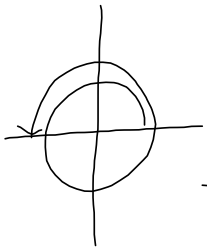
Utvider vinkelbegrepet.
Vinklene gis en retning og vi tillater flere omløp.

positiv retning

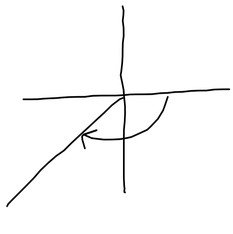




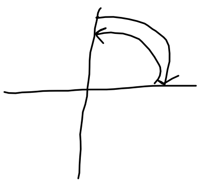
negativ retning.



vinkel
$360° + 180°$
$= 540°$



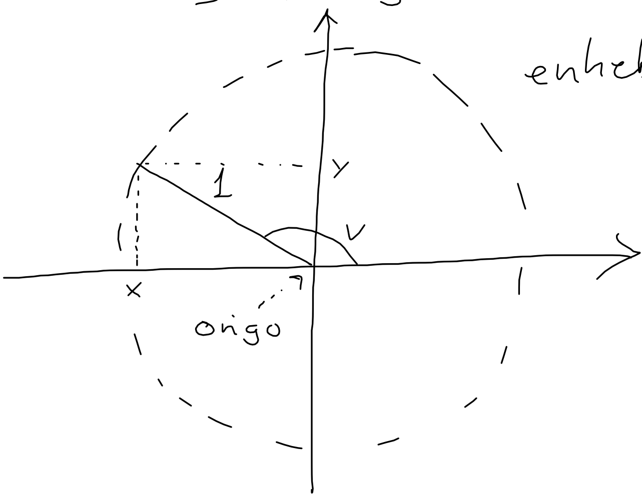
vinkelen
$-135°$



$90° + (-90°)$
$= 0°$

vinkler kan adderes og dei har inverselement.

Sin og cos for generelle vinkler



enhetssirkel radius 1
Senter i origo

Første koordinaten
$x = \cos(v)$

Andre koordinaten
$y = \sin(v)$

$180°$: $\cos 180° = -1$ $\quad \sin 180° = 0$

$270°$: $\cos 270° = 0$ $\quad \sin 270° = -1$

$-45°$: $\cos(-45°) = \frac{1}{\sqrt{2}}$ $\quad \sin(-45°) = \frac{-1}{\sqrt{2}}$

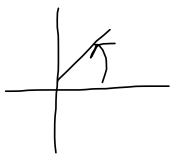
$150°$: $\cos(150°) = \frac{-\sqrt{3}}{2}$ $\quad \sin(150°) = \frac{1}{2}$

$300°$: $\cos(300°) =$ $\quad \sin(300°) =$
$\cos(-60°) = 1/2$ $\quad \sin(-60°) = -\sqrt{3}/2$

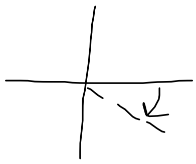
$\cos(v + 360°) = \cos(v)$
$\sin(v + 360°) = \sin(v)$

cos og sin er periodiske funksjoner.

Refleksjon om x-aksen



reflekterer

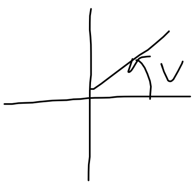


V endres til $-V$

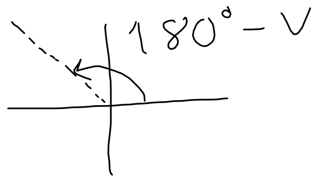
$$\cos(-V) = \cos(V)$$

$$\sin(-V) = -\sin(V)$$

Refleksjon om y-aksen



reflekterer



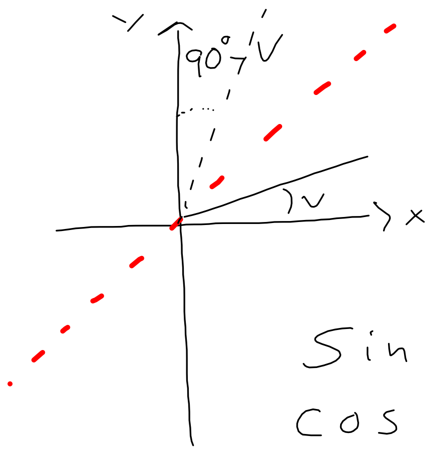
$$\sin(180^\circ - V) = \sin(V)$$

$$\cos(180^\circ - V) = -\cos(V)$$

Refleksjon om origo : (reflekterer om både x og y-aksen)
(speiling)

$$\sin(180^\circ + V) = -\sin(V)$$

$$\cos(180^\circ + V) = -\cos(V)$$



Refleksjon om linjen
$x = y$

x og y - koordinatene
bytter plass.

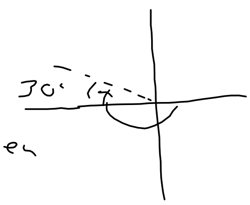
$$\sin(90° - v) = \cos v$$
$$\cos(90° - v) = \sin v$$

Dette utvider relasjonen vi fant tidligere fra $v \in [0, 90°]$ til alle v.

eks.

$\cos(-210°) = \cos(-210° + 360°)$

$= \cos(150°)$ reflektere om y-aksen



$= -\cos(180° - 150°) = -\cos(30°)$

$= \underline{-\sqrt{3}/2}$

$$\tan(v) = \frac{\sin(v)}{\cos(v)}$$

definert når
$\cos(v) \neq 0$

$v \neq 90° + 180° \cdot n$
n heltall.

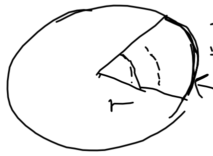
$$\tan(v + \pi) = \tan v$$

$$\left(= \frac{\sin(v+\pi)}{\cos(v+\pi)} = \frac{-\sin(v)}{-\cos(v)} = \frac{\sin(v)}{\cos(v)} \right)$$

$$\tan\left(\frac{\pi}{2} - v\right) = \frac{\cos(v)}{\sin(v)} = \frac{1}{\tan(v)}$$

$\tan(v) \neq 0$

Radianer

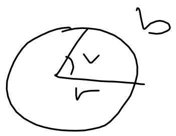


$$\frac{\text{buelengde}}{\text{radius}} = \frac{b}{r}$$ uavhengig av r.

enhetsløst

Absolutt vinkelmål er definert som $\frac{\text{buelengde}}{\text{radius}}$

vi kaller vinkelmålet radianer.



$b = r \cdot v$

Helt omløp: $360° = \frac{2\pi r}{r} = 2\pi$ (radianer)

halvt omløp: $180° = \pi$

$90° = \pi/2$

$45° = \pi/4$

$30° = \pi/6$

$60° = \pi/3$

$V_{grader} = \frac{180°}{\pi} \cdot V_{rad} \sim 57.3° \cdot V_{rad}$

$V_{rad} = \frac{\pi}{180°} V_{grader}$

1 rad $\sim 57.3^\circ$

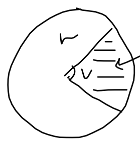
Test for å sjekke om kalkulator er stilt inn på grader eller radian

$\cos(1)$
- radianer : gir ~ 0.5
- grader : gir ~ 1

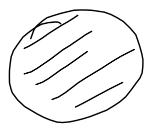
Hva er $\frac{7}{3}\pi$ rad i grader? 420°

$\left(\frac{7}{3}\pi = \left(2+\frac{1}{3}\right)\pi = 360^\circ + 60^\circ = 420^\circ\right)$

Areal til sirkelsegment:



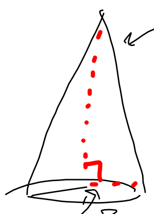
areal A



$A = \pi \cdot r^2$

$$A = \frac{b}{r}\frac{\pi r^2}{2\pi} = \underline{\underline{\frac{b \cdot r}{2}}} = \frac{v \cdot r^2}{2}$$

Kjegle



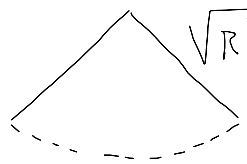
areal:

h

R

bunnplate: Areal $\pi \cdot R^2$

klipper og bretter ut



$\sqrt{R^2+h^2}$

$2\pi R$

(omkretsen til  R)

overflatearealet til en åpen kjegle er $\frac{\sqrt{R^2+h^2} \cdot 2\pi R}{2}$

$= \underline{\pi R \sqrt{R^2+h^2}}$

Lukka kjegle (legger til bunnplaten)

$A = \pi R\sqrt{R^2+h^2} + \pi R^2$