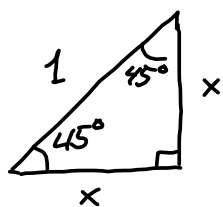


Pythagoras $a^2 + b^2 = c^2$
 $\cos(v) = \frac{a}{c}$
 $\sin(v) = \frac{b}{c}$

10.6 Eksakte verdier til sin og cos



likebeinet
trekant.

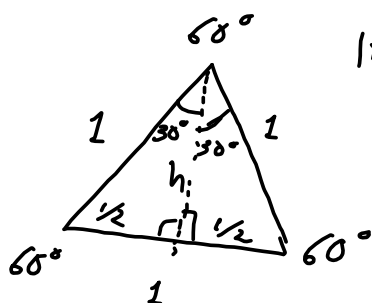
Pythagoras gir
 $x^2 + x^2 = 2x^2 = 1$

så $x^2 = \frac{1}{2}$

$x > 0$ så $x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$= \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin(45^\circ) = \cos(45^\circ) = \frac{1}{\sqrt{2}} \sim 0.707\dots$



likesidet
trekant

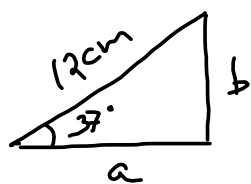
$\cos(60^\circ) = \frac{1}{2} = \sin(30^\circ)$

Pythagoras: $(\frac{1}{2})^2 + h^2 = 1^2$

$h^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$h > 0$ så $h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \sim 0.866\dots$

$\cos(30^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$

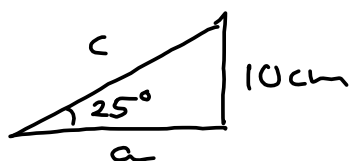


Hva er lengden på a og b?

$$\frac{a}{12\text{cm}} = \cos(37^\circ)$$

$$a = 12\text{cm} \cdot \cos(37^\circ) \sim 9.58\text{cm}$$

$$b = 12\text{cm} \cdot \sin(37^\circ) \sim 7.22\text{cm}$$



Hva er lengden på a og c?

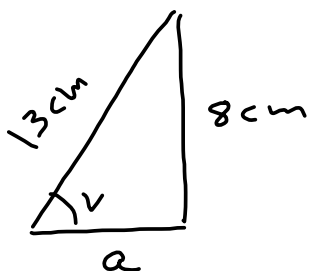
$$\sin(25^\circ) = \frac{10\text{cm}}{c} \quad \left(\times \frac{c}{\sin(25^\circ)} \right)$$

$$c = \frac{10\text{cm}}{\sin(25^\circ)} \sim 23.66\text{cm}$$

$$a = \sqrt{c^2 - (10\text{cm})^2} \dots \text{ (Pytagoras)}$$

alternativt

$$a = c \cdot \cos(25^\circ) \sim 21.44\text{cm}$$

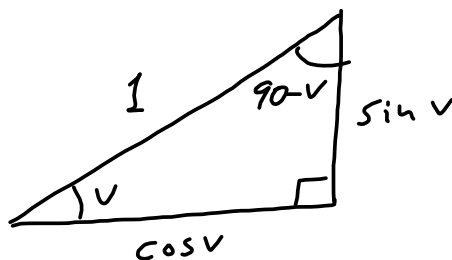


Hvor stor er vinkelen v?

$$\sin(v) = \frac{8}{13}$$

$$v = \sin^{-1}\left(\frac{8}{13}\right) = \arcsin\left(\frac{8}{13}\right)$$

$$= 37.979\dots \sim 38^\circ$$



Pytagoras: $\cos^2 v + \sin^2 v = 1$ for alle v
($\cos^2 v = (\cos v)^2$ skrivemåte.)

$$\sin(90^\circ - v) = \cos(v)$$

$$\cos(90^\circ - v) = \sin(v)$$

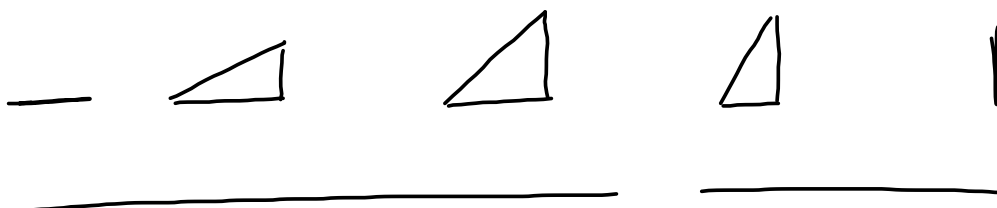
9.4 Tangens funktionen

$$\begin{aligned}\tan(v) &= \frac{\sin(v)}{\cos(v)} & \cos(v) &\neq 0 \\ &= \frac{\text{motstående katet}}{\text{hosliggende katet}}\end{aligned}$$

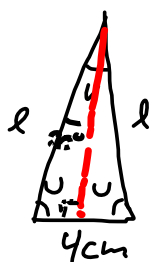
Hvis $0 \leq v < 90^\circ$, da har $\tan(v)$ verdimerings
[0, ∞)
(alle ikke-negative reelle tall.)

Eksakte verdier

v	0°	30°	45°	60°	90°
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} \sim 0.707$	$\frac{\sqrt{3}}{2}$	1
$\cos v$	1	$\frac{\sqrt{3}}{2} \sim 0.866$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan v$	0	$\frac{1}{\sqrt{3}} \sim 0.577$	1	$\sqrt{3} \sim 1.73$	ikke definert

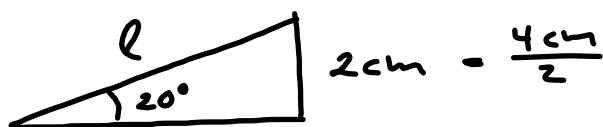


$v = 40^\circ$ Hva er lengden l ?



$$2u + v = 180^\circ$$

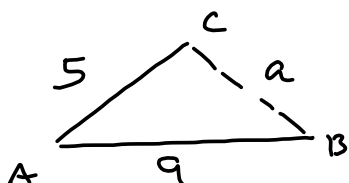
$$u = \frac{1}{2}(180^\circ - 40^\circ) = 70^\circ$$



$$2 \text{ cm} = \frac{4 \text{ cm}}{2}$$

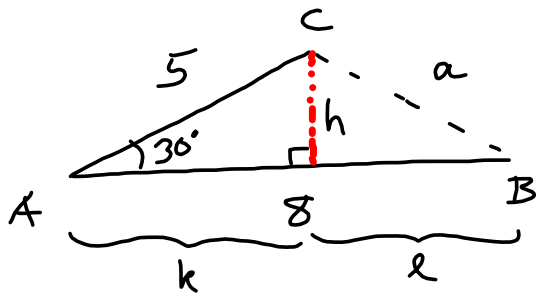
$$\sin(20^\circ) = \frac{2 \text{ cm}}{l}$$

$$l = \frac{2 \text{ cm}}{\sin(20^\circ)} = \underline{5.84 \text{ cm}}$$



Hva er lengden a ?
 a kan ikke bestemmes
 entydig.

Vi kan finne trekanter for alle verdier
 a slik at $4 = 9 - 5 < a < 14 = 9 + 5$



Hva er lengden a
Hva er vinkelene B og C?

$$k + l = 8 \text{ cm}$$

$$\frac{1}{2} = \sin(30^\circ) = \frac{h}{5 \text{ cm}} \quad h = \frac{5 \text{ cm}}{2} = 2.5 \text{ cm}$$

$$k = 5 \text{ cm} \cdot \cos(30^\circ) = 5 \text{ cm} \cdot \frac{\sqrt{3}}{2}$$

$$l = 8 \text{ cm} - k = 8 \text{ cm} - 5 \cdot \frac{\sqrt{3}}{2} \text{ cm}$$

$$a^2 = h^2 + l^2 \quad a > 0$$

$$a = \sqrt{h^2 + l^2} \approx \underline{4.4 \text{ cm}}$$

$$\tan(\angle B) = \frac{h}{l} = \frac{2.5 \text{ cm}}{(8 - 5 \cdot \frac{\sqrt{3}}{2}) \text{ cm}} = 0.6812 \dots$$

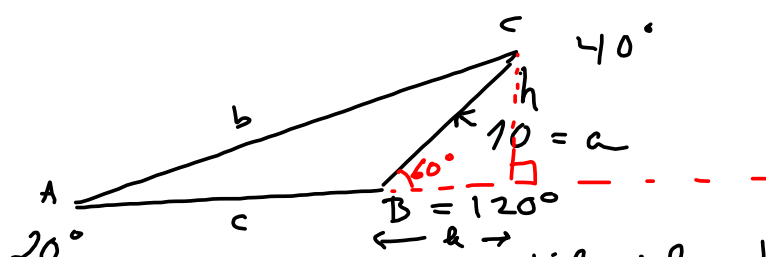
$$\text{Så} \quad \angle B = \tan^{-1}(0.6812) = \arctan(0.6812)$$

$$\underline{\angle B \sim 34^\circ}$$

$$\angle C = 180^\circ - (\angle A + \angle B)$$

$$= 180^\circ - 30^\circ - 34^\circ$$

$$\underline{\angle C = 116^\circ}$$



Finn lengdene til side b og c

$$h = 10 \cdot \sin 60^\circ = 8.66$$

$$\sin(\angle A) = \frac{h}{b}$$

$$\text{så } b = \frac{h}{\sin(\angle A)} = \frac{8.66}{\sin(20^\circ)} = \underline{25.3}$$

$$c = b \cos(\angle A) - k = b \cos(\angle A) - 10 \cdot \cos(60^\circ)$$

$$= b \cdot \cos(20^\circ) - 5 = \underline{18.8}$$