$$\frac{d}{dx} e^x = e^x$$

euler tallet

$e = 2.718281828\ldots$

irrasjonalt tall

(ikke en brøk)

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$e^{\ln x} = x \qquad x > 0$$

$$\ln e^x = x \qquad \text{alle } x$$

Eksempel

$$\frac{d}{dx} \ln x^{13} = \frac{d}{dx} 13 \ln x$$

$$= 13 \frac{d}{dx} \ln x = \frac{13}{x}$$

$$\frac{d}{dx} (\ln x)^4$$

ytre funksjon u^4

kjernen $u = \ln x$

$$= 4(\ln x)^3 \cdot (\ln x)' = \underline{\frac{4(\ln x)^3}{x}}$$

$$\mathrm{Log}(x) = \frac{\ln(x)}{\ln(10)}$$

så $\frac{d}{dx} \mathrm{Log}(x) = \frac{1}{\ln(10)} \cdot \frac{1}{x} \sim \frac{1}{2.3025} \cdot \frac{1}{x}$

$\ln(10) \cdot \mathrm{Log}(e) = 1$ Hvorfor?

$\ln(-x)$ def. for $x < 0$

$$\frac{d}{dx} \ln(-x) \overset{\text{kjerne}}{=} \frac{1}{-x} \cdot (-x)' = \frac{1}{-x} \cdot (-1)$$

$$= \frac{-1}{-x} = \underline{\frac{1}{x}}$$

$\ln|x| \qquad x \neq 0$

og $\frac{d}{dx}\ln|x| = \frac{1}{x}$

absoluttverdien til x $\quad |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Deriver $f(x) = \ln\left|\frac{1+x}{1-x}\right|$ $\quad$ defined for $x \neq -1, 1$

$= \ln|1+x| - \ln|1-x|$

$$\frac{d}{dx}f(x) = \frac{d}{dx}\ln|1+x| - \frac{d}{dx}\ln|1-x|$$

$$= \frac{1}{1+x}\cdot\underbrace{(1+x)'}_{1} - \frac{1}{1-x}\underbrace{(1-x)'}_{-1}$$

$$= \frac{1}{1+x} + \frac{1}{1-x}$$

felles nevner: $\frac{1-x}{(1+x)(1-x)} + \frac{1+x}{(1-x)(1+x)} = \frac{2}{-x^2+1}$

$$= \underline{\underline{\frac{-2}{x^2-1}}}$$

Deriver $\quad \ln|3-2x|$

$$\frac{d}{dx}\ln|3-2x| = \frac{d}{du}\ln|u| \cdot \frac{d}{dx}\overbrace{(3-2x)}^{u}$$

$$= \frac{1}{u}\cdot(-2)$$

$$= \underline{\underline{\frac{-2}{3-2x}}}$$

Løs likningen: $\ln\left(\frac{x^3}{10}\right) = \ln(\sqrt{x}) + \frac{1}{3}\ln(10)$

$\Leftrightarrow e^{\ln(x^3/10)} = e^{\ln(\sqrt{x})} \cdot e^{\frac{1}{3}\ln 10}$ (benytter $e^{a+b} = e^a \cdot e^b$)

$$\frac{x^3}{10} = \sqrt{x} \cdot 10^{1/3}$$

$$x^3 = x^{1/2} \cdot 10 \cdot 10^{1/3}$$

$$x^3 \cdot x^{-1/2} = 10^1 \cdot 10^{1/3} = 10^{1+1/3} = 10^{4/3}$$

$$x^{3-1/2} = x^{5/2} = 10^{4/3}$$

$$x = \left(x^{5/2}\right)^{2/5} = \left(10^{4/3}\right)^{2/5} = \underline{\underline{10^{8/15}}}$$

Kurvedrøfting

$$f(x) = \frac{\ln x}{x} \qquad x > 0$$

$$f'(x) = \left(\frac{\ln x}{x}\right)' = \left(\ln x \cdot \frac{1}{x}\right)' \qquad \left(\frac{1}{x} = x^{-1}\right)$$

produkt regelen

$$= (\ln x)' \cdot \frac{1}{x} + (\ln x) \cdot \left(\frac{1}{x}\right)'$$

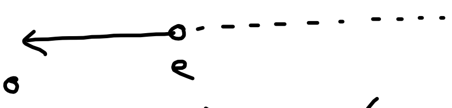
$$= \frac{1}{x} \cdot \frac{1}{x} + \ln x \cdot \frac{-1}{x^2}$$

$$= \underline{\frac{1 - \ln x}{x^2}} \qquad x > 0$$

$$f'(x) = 0 \Leftrightarrow 1 - \ln x = 0 \Leftrightarrow \ln x = 1$$

$$\Leftrightarrow x = e^{\ln x} = e^1 = e$$

Fortegnsskjema f'



maksimumspunkt $(e, f(e)) = \underline{\left(e, \frac{1}{e}\right)} = (2.7, 04)$

$\lim_{x \to 0} \frac{\ln x}{x} = -\infty$ vertikal asymptote $x = 0$ (y-aksen)

$$\lim_{x\to\infty} \frac{\ln x}{x} \; ?$$

prøver med $x = 10^n$

$$\frac{\ln 10^n}{10^n} = \frac{n \ln 10}{10^n} \sim \frac{2.3 \cdot n}{10^n}$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = 0$$

så $y=0$ (x-aksen) er en horisontal asymptote.

Ser etter vendepunkt:

$$f''(x) = \left(\frac{1-\ln x}{x^2}\right)' = \left((1-\ln x)\cdot x^{-2}\right)'$$

$$= (1-\ln x)' \cdot x^{-2} + (1-\ln x)\cdot\left(x^{-2}\right)'$$

$$= \frac{-1}{x} \cdot x^{-2} + (1-\ln x)\cdot(-2\cdot x^{-3})$$

$$= \frac{-1}{x^3} + \frac{-2(1-\ln x)}{x^3} = \frac{-3+2\ln x}{x^3}$$

$f''(x) = 0$ når $-3 + 2\ln x = 0$

$$\ln x = 3/2$$

$$x = e^{3/2} = e\cdot\sqrt{e}$$

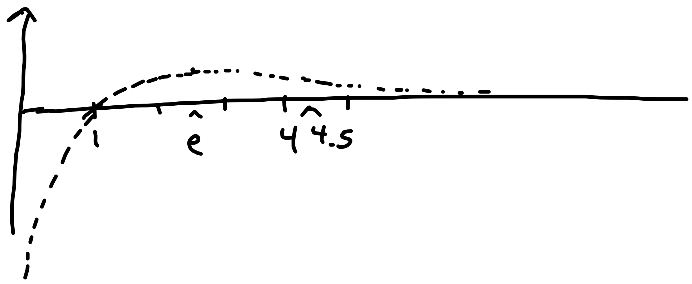
Fortegnsskjema:

Vendepunktet er $(e\sqrt{e}, f(e\sqrt{e}))$

$$\left(e\sqrt{e}, \frac{3/2}{e\sqrt{e}}\right)$$

$$\sim (4.5, 0.3)$$

nullpunktet til $f(x)$ er 1



Til orientering:

Primtallsteoremet

$$\frac{\{\overset{\text{antall}}{\#} \text{ primtall} \leq n\}}{n/\ln n} \to 1 \quad \text{når } n \to \infty$$

$$\frac{\#\text{ primtall} \leq n}{n} \sim \frac{1}{\ln n}$$

$n = 10^6$ så er $\frac{1}{\ln 10^6} = \frac{1}{6 \ln 10} \sim 0.07$

ca 7% av tallene $\leq 10^6$ er et primtall.

Renter

$10\% = 0.1$

Årlige renter (pro anno) $r_{p.a}$

P_0 pengemengde ved tiden t

$$P(n) = (1 + r_{p.a})^n P_0$$

Deler året i k like intervaller.

$$\left(1 + \frac{r}{k}\right)^k$$

La $k \to \infty$

minner om at

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$$\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^k$$

$$= \lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{\left(\frac{k}{r}\right)\cdot r} = \left(\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{k/r}\right)^r$$

$$= e^r$$

Relasjon mellom "kontinuerlig" rente og årlig rente: $e^r = 1 + r_{p.a}$