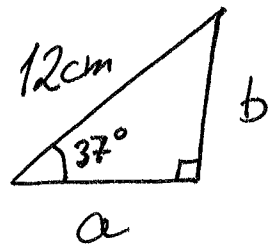
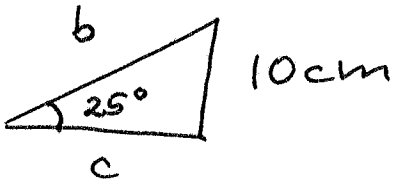
(1)



Finn a og b.
(lengden til a og b)

$b = 12\text{cm} \cdot \sin(37°) \approx 7.22\text{cm}$

$a = 12\text{cm} \cdot \cos(37°) \approx 9.58\text{cm}$



Finn b og c.

$\sin(25°) = \frac{10\,\text{cm}}{b}$

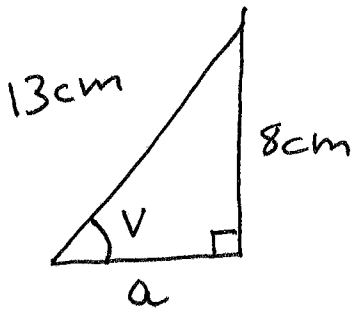
så $b = \frac{10\,\text{cm}}{\sin(25°)} \approx 23{,}66\text{cm}$

$c = b \cdot \cos(25°) \approx 21.44\text{cm}$

Alternativt kunne vi benytte Pytagoras:

$b^2 = (10\text{cm})^2 + c^2$

$c = \sqrt{b^2 - (10\text{cm})^2}$ ...



Hva er vinkelen V?
Hva er lengden til side a?

$\sin(V) = \frac{8\,\text{cm}}{13\,\text{cm}} = \frac{8}{13}$

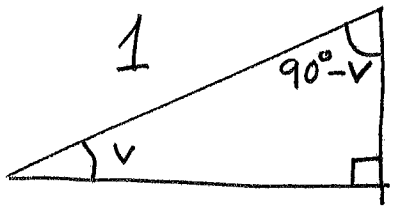
$V = \arcsin\left(\frac{8}{13}\right)$ (eller $\sin^{-1}\left(\frac{8}{13}\right)$)

$= 37.979... \approx \underline{38°}$

* $a = 13\text{cm} \cdot \cos(V)$
* Eller Pytagoras: $a^2 = (13\text{cm})^2 - (8\text{cm})^2 = (169 - 64)\text{cm}^2$
$= 105\text{cm}^2$

$a \approx 10.25\text{cm}$

(2)



Pytagoras sin sats $(\sin v)^2 + (\cos v)^2 = 1^2$

$\sin^2 v + \cos^2 v = 1$

$0° < v < 90°$

$\sin(90° - v) = \cos(v)$

$\cos(90° - v) = \sin(v)$

Tangens funksjonen

$\tan(v) = \frac{\sin(v)}{\cos(v)}$

$= \frac{\text{motstående katet}}{\text{hosliggende katet.}}$

Bare definert når: $\cos(v) \neq 0$

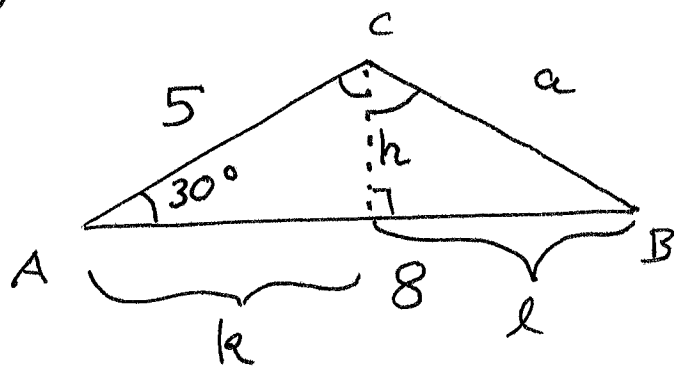
Verdimengden til $\tan(v)$ er $[0, \infty\rangle$

når $0° \leq v < 90°$

v	0°	30°	45°	60°	90°
sin v	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos v	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan v	0	$1/\sqrt{3}$	1	$\sqrt{3}$	kan ingenverdi (er ikke def.)

(Eksakte verdier står i 7.2 i boka)

③



Hva er lengden a?
Hva er vinklene B og C.

$k + l = 8$

$h = 5 \cdot \sin(30°) = 5 \cdot \frac{1}{2} = 5/2.$

$k = 5 \cdot \cos(30°) = 5 \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$

$l = 8 - k = 8 - \frac{5\sqrt{3}}{2}$

$\tan(\angle B) = \tan(B) = \frac{h}{l} = \frac{5/2}{8 - 5\sqrt{3}/2} = \frac{5}{16 - 5\sqrt{3}}$

$B = \arctan\left(\frac{h}{l}\right)$

$B = \arctan(0.6812..)$

$= 34.26... \approx 34°$ (2 gyldige siffer)

$\angle C = 180° - 30° - 34° = (180 - 64)°$

Vinkel c er $C = \underline{116°}$

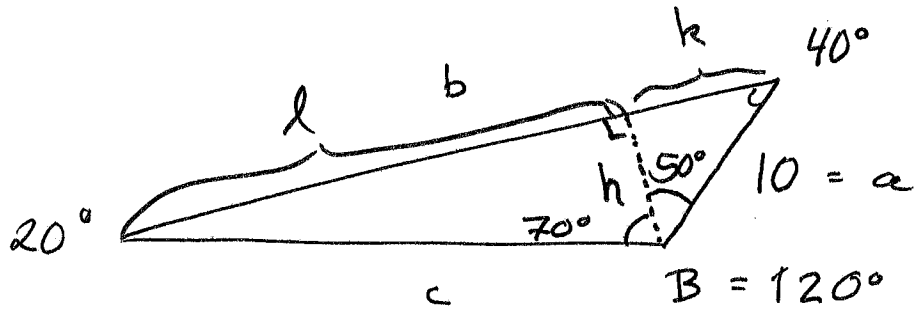
$a = \sqrt{l^2 + h^2}$ eller $a = \frac{h}{\sin(B)}$

$= \sqrt{\left(\frac{5}{2}\right)^2 + \left(8 - \frac{5\sqrt{3}}{2}\right)^2} \approx 4.4$

↑

(hvis dette benyttes bør $B = 34.26°$ brukes heller enn $34°$)

(4)



Finn lengden til b og c.

$$\sin(40°) = \frac{h}{10} \approx 6.43$$

$$h = 10 \cdot \sin(40°)$$

$$\tan(70°) = \frac{l}{h}$$

$$l = h \cdot \tan(70°) \approx \underline{17.7}$$

$$\cos(70°) = \frac{h}{c}$$

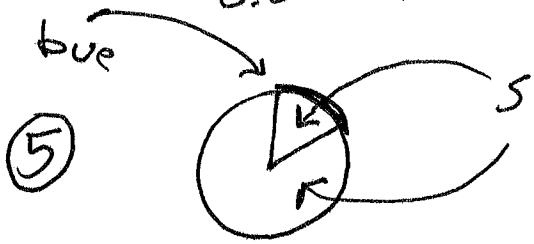
$$c = \frac{h}{\cos(70°)} \approx \underline{\underline{18.8}}$$

$$b = k + l = 10 \cdot \cos(40°) + l \approx \underline{\underline{25.3}}$$

Lengden til sidene er

$b = \underline{25.3}$ og $c = \underline{18.8}$.

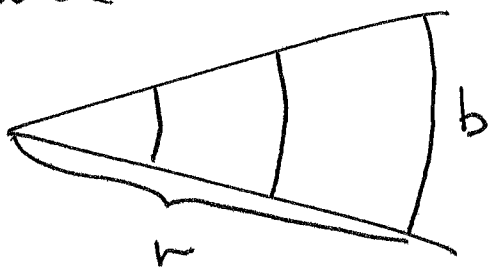
6.8 Radianer, absolutt vinkelmål



⑤

Del av en sirkel avgrenset av to linjer fra sentrum ut til randen.

Gitt vinkel



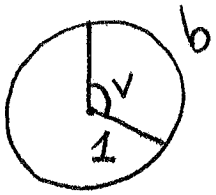
$\frac{b}{r}$ $\frac{\text{buelengde}}{\text{radius}}$

er uavhengig av r (hvilket sirkelsegment vi benytter)

$\frac{b}{r}$ er enhetsløst.

Absolutt vinkelmål er definert som $\frac{\text{buelengde}}{\text{radius}}$

som et vinkelmål gir vi det benevning (enhet) radianer.



Hvis radius er lik 1, da er vinkelen lik buelengden (med enhet rad)

Et helt omløp : $360° = \frac{2\pi \cdot r}{r} = 2\pi$ rad

(omkrets til en sirkel m. radius r er lik $2\pi r$)

$$\boxed{180° = \pi \text{ rad}}$$

(6)

$90° = \frac{\pi}{2}$ rad

$45° = \frac{\pi}{4}$ rad

$30° = \frac{\pi}{6}$ rad

$60° = \frac{\pi}{3}$ rad ≈ 1.05 rad.

$0° = 0$ rad

$\cos(1°) \approx 1$

$\cos(1\text{rad}) \approx \cos(\frac{\pi}{3}) = \frac{1}{2}$

$\cos(1) \approx 1$: DEG

$\approx \frac{1}{2}$: RAD

"test for innstilling av gradeenhet".

$V_{\text{grader}} = \frac{180°}{\pi \text{ rad}} V_{\text{rad}} \approx 57.3°/\text{rad} \cdot V_{\text{rad}}$

$V_{\text{rad}} = \frac{\pi \cdot \text{rad}}{180°} V_{\text{grad}}$

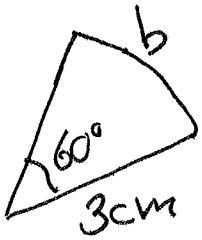
$1 \text{ rad} = \frac{180°}{\pi \cdot \text{rad}} \cdot 1 \text{ rad} = 57.3°$

6.9 Buelengde

$v = \frac{b}{r}$

$b = r \cdot v$

buelengde = radius · vinkel (enhet radianer)



Hva er buelengden?

$b = 3\text{cm} \cdot \left(\frac{\pi \text{ rad}}{180°} 60°\right) = 3\text{cm} \cdot \frac{\pi}{3}\text{rad}$

$b = \pi \text{ cm}$ (rad bortfaller)