

Løs diff. likningen

Randbetingelse

2 mai 2013  $y'(3x+2) = 2y-3$   $y(2) = 2$

Dette er en separabel diff likning

(1)  $\frac{y'}{2y-3} = \frac{1}{3x+2}$  ( $y' \cdot p(y) = q(x)$ )

$$\int \frac{1}{2y-3} dy = \int \frac{y'}{2y-3} dx = \int \frac{1}{3x+2} dx$$

Lineær substitution:

$$\int \frac{1}{2y-3} dy$$

$$u = 2y-3$$

$$du = 2dy$$

$$dy = \frac{1}{2} du$$

$$= \int \frac{1}{u} \cdot \frac{1}{2} \cdot du$$

$$= \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|2y-3| + C$$

Tilsvarende er  $\int \frac{1}{3x+2} dx = \frac{1}{3} \ln|3x+2| + C$

$$\frac{1}{2} \ln|2y-3| = \frac{1}{3} \ln|3x+2| + C$$

ganger med 2

$$\ln|2y-3| = \frac{2}{3} \ln|3x+2| + C$$

(børker:  $r \cdot \ln a = \ln a^r$ )

$$\ln|2y-3| = \ln|3x+2|^{2/3} + C$$

$$|2y-3| = |3x+2|^{2/3} \cdot e^C$$

$$2y-3 = K(3x+2)^{2/3} \quad K \in \mathbb{R}$$

$$y(x) = \frac{1}{2} \left[ 3 + K(3x+2)^{2/3} \right] \quad \left( \begin{array}{l} K=0 \\ \text{sjekkes} \\ \text{separat} \end{array} \right)$$

Randbetingelsen  $y(2) = 2$  gir

$$(2) \quad 2 = \frac{1}{2} \left[ 3 + K \left( \frac{8^{2/3}}{4} \right) \right] \quad \text{ganger med 2}$$

$$4 = 3 + 4K$$

$$4 - 3 = 1 = 4K, \quad \underline{K = \frac{1}{4}}$$

$$y(x) = \underline{\frac{1}{2} \left[ 3 + \frac{1}{4} (3x+2)^{2/3} \right]}$$

Eksempel  $y' - \frac{1}{y} = 2$

$$y' = 2 + \frac{1}{y} = \frac{2y+1}{y}$$

$$\left( \frac{y}{2y+1} \right) \cdot y' = 1$$

$$\int \frac{y}{2y+1} dy = \int dx = x + C$$

$$\frac{1}{2} \int \frac{y}{y+\frac{1}{2}} dy = \frac{1}{2} \int \frac{y+\frac{1}{2}-\frac{1}{2}}{y+\frac{1}{2}} dy$$

$$= \frac{1}{2} \int \left( 1 - \frac{1/2}{y+1/2} \right) dy$$

$$= \frac{1}{2} \left[ y - \frac{1}{2} \ln |y + \frac{1}{2}| \right] + C$$

$$\frac{1}{2} \left[ y - \frac{1}{2} \ln |y + \frac{1}{2}| \right] = x + C$$

ganger med 4

$$\underline{2y - \ln |y + \frac{1}{2}| = 4x + C}$$

Dette gir en implisitt beskrivelse av  $y(x)$ .