31.01.2013 Kvotientregelen 9.9

Neste uke: Næringslivsdag torsdag 7.02.

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- Implisitt derivasjon, kobla hastigheter. (Ikke i boka, hefte på nettsiden vår)
- Geogebra
- Test.

Derivasjonsreglene

$$\frac{d}{dx} f = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} x^r = r \cdot x^{r-1}$$

$x > 0$
r reelt tall. ...

$$\sqrt{x} = x^{1/2} \qquad \frac{1}{\sqrt[3]{x}} = \frac{1}{x^{1/3}} = (x^{1/3})^{-1} = x^{-1/3}$$

$$\sqrt[n]{x^m} = x^{m/n}$$

$$(k \cdot f(x))' = k(f'(x))$$

$$(f(x) + g(x))' = f'(x) + g'(x)$$

} Derivasjon er lineært (en lineær operasjon)

$$(f \cdot g)' = f'g + fg' \qquad \text{produktregel}$$

$$f(u(x))' = f'(u(x)) \cdot u'(x) \qquad \text{kjerneregel}$$

$$\left(\frac{d f(u(x))}{dx} = \frac{d f(u)}{du} \cdot \frac{du}{dx} \right)$$

Deriverbar $\Rightarrow$ kontinuerlig.

$f(x)$ er kontinuerlig i $x = a$ hvis $\lim_{x \to a} f(x) = f(a)$.

$$\lim_{h \to 0} f(a+h) - f(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \cdot h$$

(deriverbar gir:)

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \cdot \lim_{h \to 0} h = f'(a) \cdot 0 = 0$$

så $\lim_{h \to 0} f(a+h) = f(a)$ og $f(x)$ er kontinuerlig i $x=a$.

(til orientering)

Bevis for produktregelen:

② Δx liten endring i x "delta x"

$$f(x+\Delta x) - f(x) = \Delta f(x)$$ (Δx underforstått)

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x} \quad \left(= \frac{df}{dx}\right).$$

$$f(x+\Delta x) = f(x) + \Delta f(x)$$

$$(f \cdot g)'(x) = \lim_{\Delta x \to 0} \frac{(f+\Delta f)(g+\Delta g) - f \cdot g}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f \cdot \Delta g + \Delta f \cdot g + \Delta f \cdot \Delta g}{\Delta x}$$

$$= \lim_{\Delta x \to 0} f \cdot \frac{\Delta g}{\Delta x} + \frac{\Delta f}{\Delta x} \cdot g + \frac{\Delta f}{\Delta x} \Delta g$$

$$= f \lim_{\Delta x \to 0} \frac{\Delta g}{\Delta x} + \left(\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}\right) \cdot g + \underbrace{\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}}_{f'} \cdot \underbrace{\lim_{\Delta x \to 0} \Delta g}_{0}$$

$$(f \cdot g)' = f \cdot g' + f' \cdot g$$

$$f(x) = \frac{x+1}{x+3}$$

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$$= (x+1) \cdot \frac{1}{x+3} = (x+1) \cdot (x+3)^{-1}$$

$$f'(x) = \left((x+1)(x+3)^{-1}\right)'$$

$$= (x+1)' \cdot (x+3)^{-1} + (x+1)\left((x+3)^{-1}\right)'$$

$$= 1 \cdot \frac{1}{x+3} + (x+1) \cdot \frac{-1}{(x+3)^2} \cdot (x+3)'$$

$$= \frac{1}{x+3} - \frac{(x+1)}{(x+3)^2} = \frac{(x+3)-(x+1)}{(x+3)^2}$$

$$f'(x) = \underline{\frac{2}{(x+3)^2}}$$

Vi utfører polynomdivisjon først og deriverer etterpå:

$$f(x) = \frac{x+1}{x+3} = \frac{x+3-2}{x+3} = 1 - \frac{2}{(x+3)}$$

$$f'(x) = (1)' - 2\left((x+3)^{-1}\right)'$$

$$= 0 - 2\left(-1(x+3)^{-2}\right) = \underline{\frac{2}{(x+3)^2}}.$$

$$\left(\frac{1}{g(x)}\right)' = \left((g(x))^{-1}\right)' = \frac{-1}{(g(x))^2} \cdot g'(x)$$

$$\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{(g(x))^2}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x) \cdot \frac{1}{g(x)}\right)' = f'(x) \cdot \frac{1}{g(x)} + f(x) \cdot \left(\frac{1}{g(x)}\right)'$$

$$= \frac{f'(x)}{g(x)} + \frac{-f(x) \cdot g'(x)}{g^2(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

(fellesnevner)

4) Kvotientregelen $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Vi bruker kvotientregelen til å finne den deriverte til $f(x) = \frac{x+1}{x+3}$.

$$f'(x) = \left(\frac{x+1}{x+3}\right)' = \frac{(x+1)'(x+3) - (x+1)(x+3)'}{(x+3)^2}$$

$$= \frac{(x+3) - (x+1)}{(x+3)^2} = \underline{\underline{\frac{2}{(x+3)^2}}}$$

$$g(x) = \frac{1}{(x^2+2)^5} = (x^2+2)^{-5}$$

$$g'(x) = -5(x^2+2)^{-6}(x^2+2)'$$

$$= \underline{-10x(x^2+2)^{-6}}.$$

Kvotientregelen: $\left(\frac{1}{((x^2+2)^5)}\right)' = -\frac{((x^2+2)^5)' + \overbrace{(1)'}^{0}(x^2+2)^5}{((x^2+2)^5)^2}$

$$= -\frac{5(x^2+2)^4(x^2+2)'}{(x^2+2)^{10}}$$

$$= -10x \cdot \frac{(x^2+2)^4}{(x^2+2)^{10}} = -10x\frac{1}{(x^2+2)^6} = \underline{-10x(x^2+2)^{-6}}$$

ex $f(x) = \frac{\sqrt{x+1}}{x}$

Finn $f'(x) = \frac{\overbrace{(\sqrt{x+1})'}^{(x+1)^{1/2}} \cdot x - \sqrt{x+1}(x)'}{(x)^2}$

$$= \frac{\left(\frac{1}{2\sqrt{x+1}} \cdot x - \sqrt{x+1} \cdot 1\right)}{x^2} \frac{2\sqrt{x+1}}{2\sqrt{x+1}}$$

$$= \frac{x - 2(x+1)}{2x^2\sqrt{x+1}} = \underline{\underline{\frac{-x-2}{2x^2\sqrt{x+1}}}}$$

(5)

$$f(x) = \frac{\sqrt{2x+1}}{x^7} = \sqrt{2x+1} \cdot x^{-7}$$

$$f'(x) = \left(\sqrt{2x+1}\right)' \cdot x^{-7} + \sqrt{2x+1} \cdot \left(x^{-7}\right)'$$

$$= \left(\frac{1}{2}(2x+1)^{-1/2} \cdot (2x+1)'\right) \cdot x^{-7} + \sqrt{2x+1}\,(-7)\,x^{-8}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{2x+1}} \cdot x^{-7} + (-7)\sqrt{2x+1} \cdot x^{-8}$$

$$= \frac{1}{x^7\sqrt{2x+1}} - \frac{7\sqrt{2x+1}}{x^8}$$

$$= \frac{1}{x^8\sqrt{2x+1}}\left(x - 7(2x+1)\right)$$

$$= \frac{-13x-7}{x^8\sqrt{2x+1}}$$

$$f(x) = \frac{x^3 - x^2 - 2x}{x^2 - 2x} = \frac{x^2 - x - 2}{x-2}$$

$$= \frac{(x-2)(x+1)}{x-2} = x+1$$

$$f'(x) = (x+1)' = 1.$$

ex

$$f(x) = \frac{x^4 - 2x\sqrt{x}}{\sqrt{x}} = \frac{x^4}{\sqrt{x}} - 2\frac{x\sqrt{x}}{\sqrt{x}}$$

$$= x^4 \cdot x^{-1/2} - 2x = x^{7/2} - 2x$$

$$f'(x) = \left(x^{7/2}\right)' - (2x)' = \frac{7}{2}x^{5/2} - 2$$

$$= \frac{7}{2}x^2\sqrt{x} - 2$$

(6) Deriver $f(x) = \dfrac{\sqrt{x}}{x^2+1} \quad \left(= \dfrac{1}{x^2/\sqrt{x} + 1/\sqrt{x}} = \dfrac{1}{x^{3/2} + x^{-1/2}} \dots \right)$

Kvotientregelen :

$$f'(x) = \frac{(\sqrt{x})'(x^2+1) - \sqrt{x}(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{\left(\frac{1}{2\sqrt{x}}\right)(x^2+1) - \sqrt{x} \cdot 2x}{(x^2+1)^2} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{(x^2+1) - 2x \cdot 2x}{2\sqrt{x}(x^2+1)^2}$$

$$= \underline{\underline{\frac{1-3x^2}{2\sqrt{x}(x^2+1)^2}}}$$