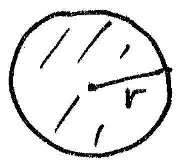
30.04.2012

Volum 16.5 - 6

(1)

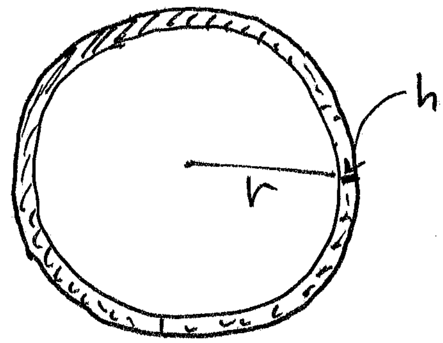


Arealet til en sirkel $A(r) = \pi r^2$

Omkretsen til en sirkel $O(r) = 2\pi r$.

$$\frac{d}{dr} A(r) = O(r)$$

$$\overset{def}{=} \lim_{h \to 0} \frac{A(r+h) - A(r)}{h}$$



$$h\, O(r) < A(r+h) - A(r) < h\, O(r+h)$$

$h > 0$ $$O(r) < \frac{A(r+h) - A(r)}{h} < O(r+h)$$

$O(r)$ er en kontinuerlig funksjon.

Tar grensen $h \to 0^+$

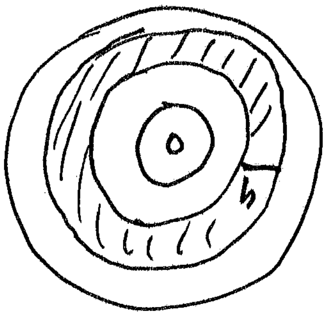
$$O(r) \leq \frac{dA(r)}{dr} \leq O(r) \quad \left(= \lim_{h \to 0^+} O(r+h)\right)$$

Derfor er $\underline{O(r) = \frac{dA(r)}{dr}}$.

"Motsatt vei" Integrerer

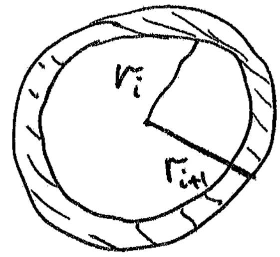
Radius R

(2)



$0 < r_1 < r_2 < \dots < r_n = R$

$$\sum_{i=0}^{n-1} \underbrace{\underset{\text{tykkelse}}{(r_{i+1} - r_i)}\, O(r_i)}_{\text{tilnærmer areal}}$$



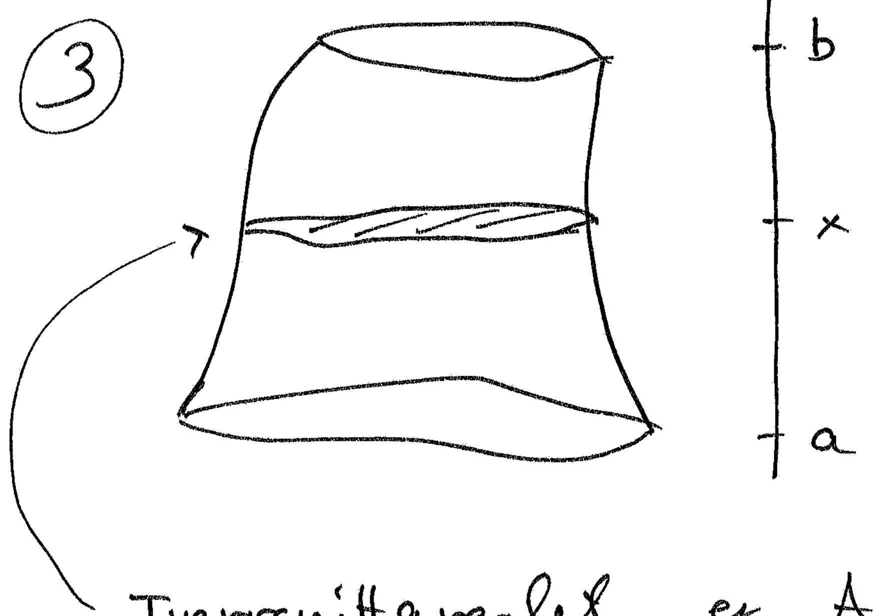
Tar grensen hvor $(r_{i+1} - r_i) \to 0 \quad (n \to \infty)$

$$\int_0^R O(r)\,dr = A(R).$$

$$\left(\int_0^R 2\pi r\,dr = \pi r^2 \Big|_0^R = \pi R^2. \right)$$

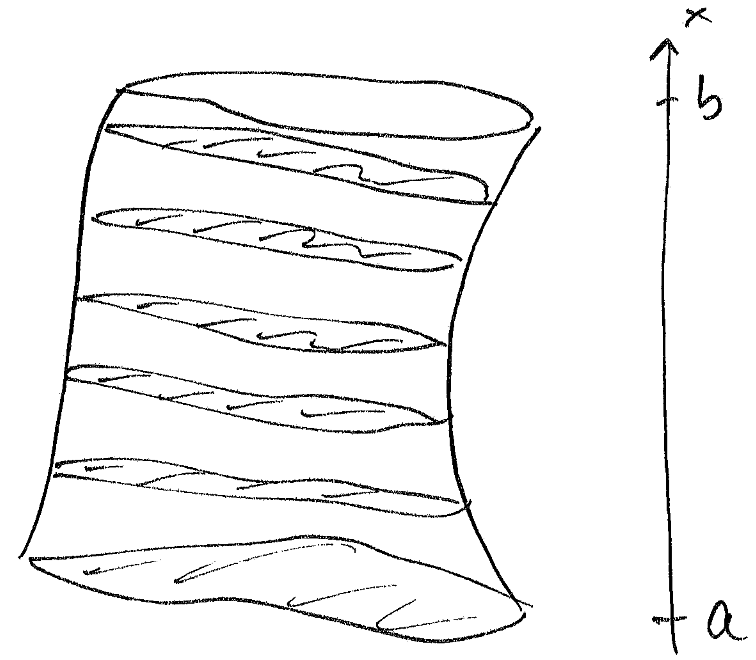
Legeme

3



Tverrsnittarealet er $A(x)$

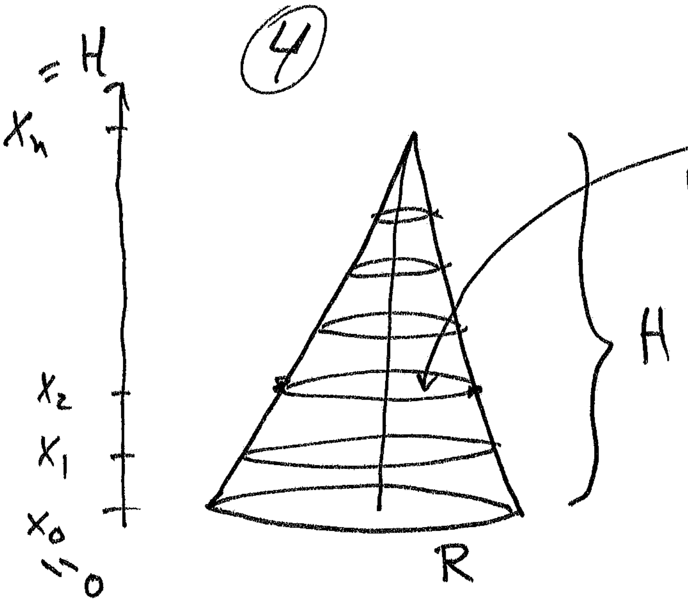
Volumet til legemet er $\underline{V = \int_a^b A(x)\,dx}$.



$$V \sim \sum_{i=0}^{n-1} \underbrace{(x_{i+1} - x_i)}_{\text{høyden}} A(x_i)$$

4

$$r(x) = R - \frac{R}{H} \cdot x$$



radius
til tverrsnittdisken $r(x)$.

Deler kjeglen i n like høye biter.
Høyden til hver bit $\frac{H}{n}$.

$$x_i = \frac{i \cdot H}{n}.$$

$$\pi x_i^2 \frac{H}{n} \leq \text{Volum } i\text{-te del} \leq \pi x_{i-1}^2 \cdot \frac{H}{n}$$

summerer fra $i=1$ til $i=n$:

$$\underbrace{\frac{\pi H}{n}\left(r(x_1)^2 + \cdots + r(x_n)^2\right)}_{G_1} \leq \text{Volumet til kjeglen} \leq \underbrace{\frac{\pi H}{n}\left(r(x_0)^2 + \cdots + r(x_{n-1})^2\right)}_{G_2}$$

$$G_2 - G_1 = \pi H \left(r(x_0)^2 - r(x_n)^2\right) \cdot \frac{1}{n} = \pi H R^2 / n$$

Dette går mot 0 når $n \to \infty$.

$$\pi \left(r(x_0)^2 + \cdots + x_{n-1}^2\right) \cdot \left(\frac{H}{n}\right)^{\to dx} \longrightarrow \pi \int r(x)^2 \, dx$$

$$\pi \int_0^H \left(R - \frac{R}{H}x\right)^2 dx =$$

$U = R - \frac{R}{H}x$.

$dU = -\frac{R}{H} dx$

Substitusjon

⑤

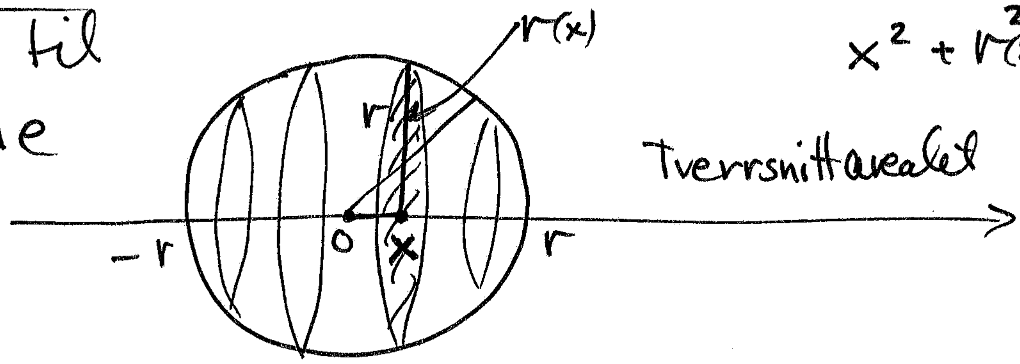
$$\pi \cdot \int_R^0 U^2 \left(\frac{-H}{R} dU\right)$$

$$= \pi \int_0^R \frac{H}{R} U^2 dU = \pi \frac{H}{R} \frac{U^3}{3}\Big|_0^R$$

$$= \pi \frac{H}{R}\left(\frac{R^3}{3} - 0\right) = \frac{\pi}{3} H R^2$$

$$= \underline{\frac{(\pi R^2) \cdot H}{3}}$$

Volum til en kule



Pytagoras

$x^2 + r^2_{(x)} = r^2$

Tverrsnittarealet $A(x) = \pi \cdot r^2_{(x)}$

$= \underline{\pi(r^2 - x^2)}$

r radius til kulen

$r(x)$ radius til tverrsnittdisken for x

$$V = \int_{-r}^{r} A(x) \cdot dx = \int_{-r}^{r} \pi(r^2 - x^2) dx =$$

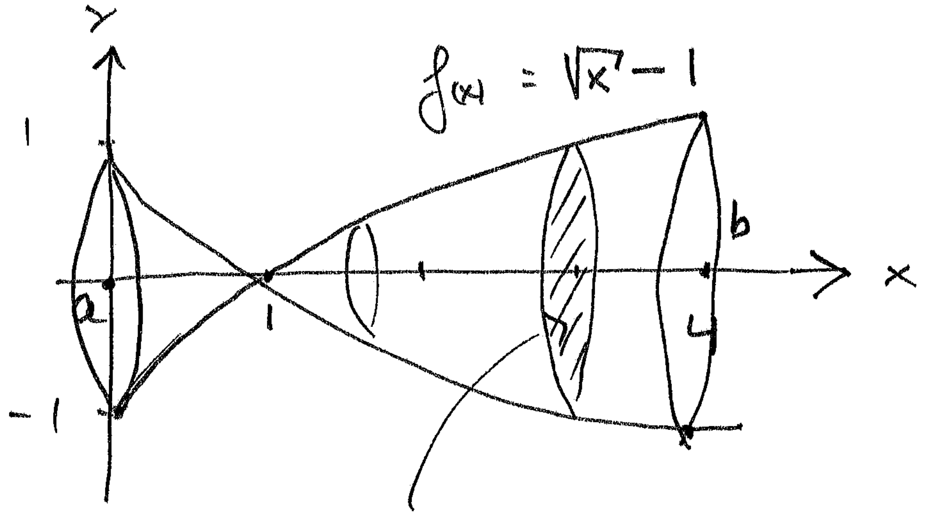
$$2\int_0^r \pi(r^2 - x^2) dx = 2\pi\left(r^2 \cdot x - \frac{x^3}{3}\right)\Big|_0^r = 2\pi\left(r^2 \cdot r - \frac{r^3}{3}\right)$$

$$= 2\pi \frac{2r^3}{3} = \underline{\frac{4\pi r^3}{3}}$$

(6)

Omdreininglegemer 16.6

$a = 0$
$b = 4$



Sirkel med radius $f(x)$.

Tverrsnitt arealet $A(x) = \pi (f(x))^2$.

Volumet $V = \int_a^b A(x) dx$

$$= \int_a^b \pi f^2(x) \, dx$$

$$V = \pi \int_a^b f^2(x) \, dx$$

$$V = \pi \int_0^4 (\sqrt{x} - 1)^2 dx = \pi \int_0^4 x - 2\sqrt{x} + 1 \, dx$$

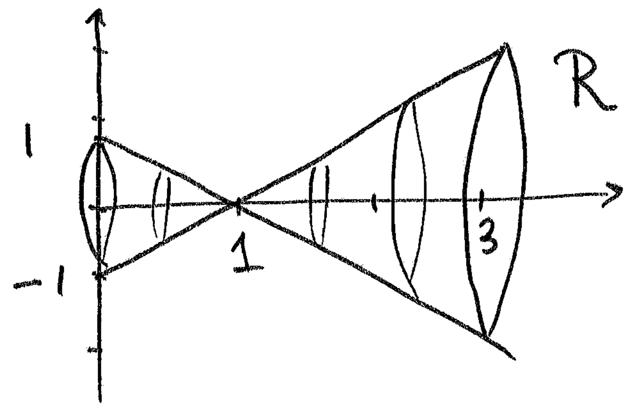
$$= \pi \left[\frac{x^2}{2} - 2\left(\frac{x^{3/2}}{3/2}\right) + x \right]_0^4$$

$$= \pi \left[8 - \frac{4}{3}\underbrace{(4 \cdot 2)}_{4^{3/2}} + 4 \right] = \pi \left[12 - \frac{32}{3} \right]$$

$$= \pi \frac{4}{3} = 4.18$$

oppg. $f(x) = x - 1$ for $x \in [0,3]$.

(7) Finn volumet til regionen R gitt som omdreiningslegemet til $f(x)$ for $x \in [0,3]$. (rundt x-aksen)



$$V = \int_0^3 \pi f^2(x)\, dx = \int_0^3 \pi (x-1)^2\, dx$$

La $u = x - 1$
$du = dx$

$$= \int_{-1}^{2} \pi u^2\, du$$

$$= \pi \frac{u^3}{3}\Big|_{-1}^{2} = \frac{\pi}{3}\left(2^3 - (-1)^3\right) = \frac{\pi}{3}(8-(-1))$$

$$= \underline{\underline{3\pi}}$$

R består av to kjegler.

Volumet til den lille kjeglen : $\frac{\pi}{3} \cdot 1 \cdot (1)^2 = \frac{\pi}{3}$

——— store ——— : $\frac{\pi}{3} \cdot 2 \cdot (2)^2 = \frac{8\pi}{3}$.

Summen er $\underline{3\pi}$

oppgave til torsdag.

(8) En sylinder med radius 1 bores ut av en kule med radius 2.
(midt i kulen).
Hva er volumet til det som er igjen av kulen?