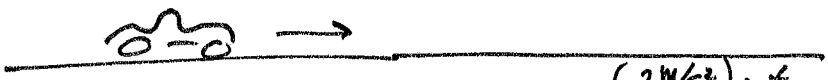
12.04.2012 15.7 og 15.8

(1) En bil kjører langs en rett strekning



$(2\,\mathrm{m/s^2})\cdot t$

og farten er gitt ved $V(t) = 2\cdot t$

$0\,\mathrm{s} \le t \le 9\,\mathrm{s}$

Hva er strekningen som biten tilbakelegger på de 9 sekundene?

$S(t)$ posisjon ved tiden $t$.

$$S'(t) = V(t)$$

$$\int_0^9 V(t)\,dt = S(9) - S(0)$$

er tilbakelagt strekning.

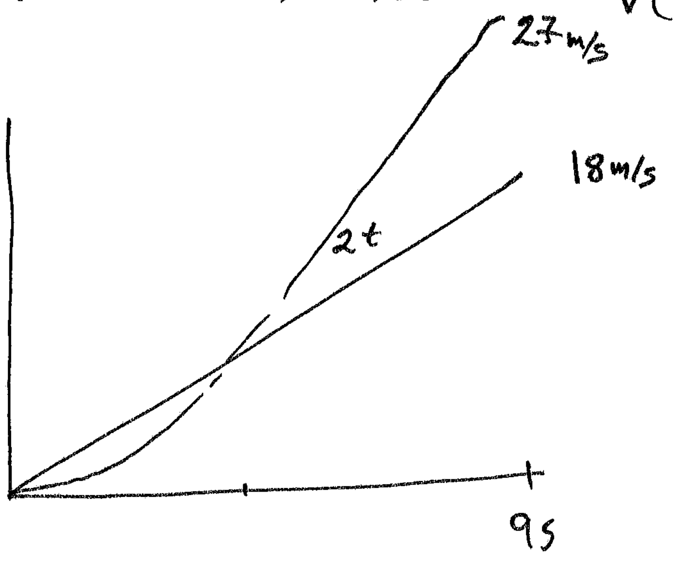
Det er: $\int_0^9 2\cdot t\,dt$

$$= t^2 \Big|_0^9 = 9^2 - 0^2$$

$$= \underline{81\ \mathrm{m}}$$

oppg. Finn strekningen bilen tilbakelegger (på 9 sekunder) når $V(t) = t^{3/2} = t\cdot\sqrt{t}$

$$(1\,\mathrm{m/s^{5/2}})\cdot t^{3/2}$$



Strekningen $\quad S(9) - S(0)$

(2)

$$= \int_0^9 V(t)\,dt$$

$$= \int_0^9 t^{3/2}\,dt$$

$$= \frac{t^{3/2+1}}{\frac{3}{2}+1} \Big|_0^9$$

$$= \frac{t^{5/2}}{5/2} \Big|_0^9$$

$$= \frac{2}{5} t^{5/2} \Big|_0^9$$

$$= \frac{2}{5}\left( (9^{5/2}) - 0^{5/2} \right) \qquad \left(\text{siden } 9^{1/2} = 3\right)$$

$$= \frac{2}{5}\left(3^5\right)$$

$$= \frac{2}{5}\, 3 \cdot 81 = \underline{97.2\,\text{m}}.$$

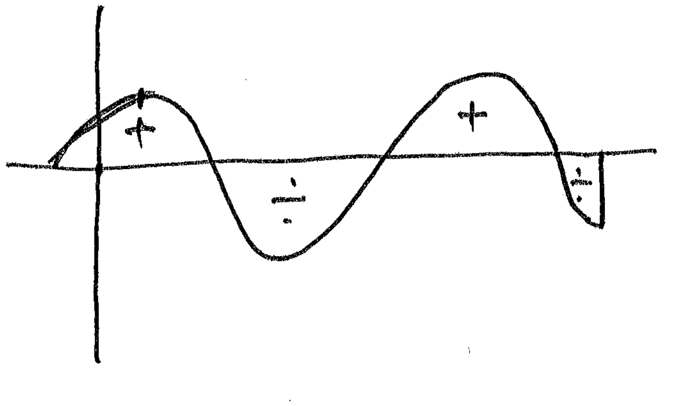
—

oppg $\quad \displaystyle\int_0^{\pi} \sin(3x+1)\,dx$

$$-\frac{\cos(3x+1)}{3} \Big|_0^{\pi}$$

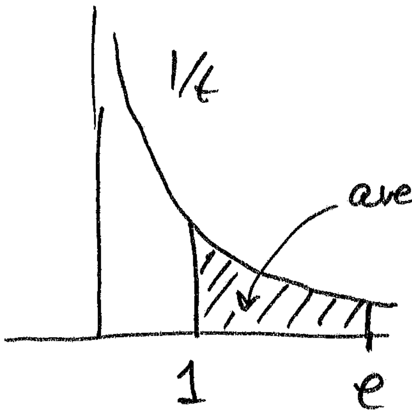
$$= \frac{1}{3}\left[ -\cos(3\pi+1) + \cos(1) \right]$$

$$= 0.36\ldots$$



③ $\int_1^x \frac{1}{t}\,dt = \ln t \Big|_1^x = \ln x - \ln 1$

$= \underline{\ln x}$. $\qquad x > 0.$

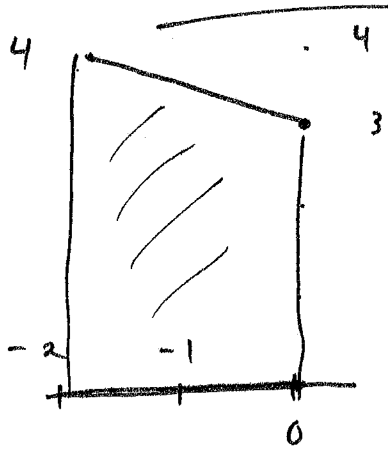



$\int_x^1 \frac{1}{t}\,dt \qquad 0 < x < 1$

$= -\int_1^x \frac{1}{t}\,dt = -\ln x$

$\lim_{x \to 0^+} -\ln x = +\infty.$

$\int_0^1 \frac{1}{x}\,dx = \lim_{x \to 0^+} \int_x^1 \frac{1}{t}\,dt$ <u>eksisterer ikke</u>.




— Merk at $\underline{-2 < 0}$

$\int_0^{-2} 3 - \frac{x}{2}\,dx =$

$= -\int_{-2}^{0} 3 - \frac{x}{2}\,dx$

$= \underline{-7}$ (fra figuren)

(Husk at $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$)

Alternativt (fundamentalteoremet)

$-\left[3x - \frac{x^2}{4}\right]_{-2}^{0} = 3(-2) - \frac{(-2)^2}{4} = \underline{-7}$

Når eksisterer

(4) $$\int_0^1 \frac{1}{t^r}\,dt = \lim_{x\to 0^+} \int_x^1 \frac{1}{t^r}\,dt \quad ?$$

$r=1$ eksister ikke.

$r>1$ $\quad t^r < t \quad\quad 0<t<1$

$$\frac{1}{t^r} > \frac{1}{t}$$

Så $\int_0^1 \frac{1}{t^r}\,dt$ eksister ikke for $r \geq 1$.

$r=\frac{1}{2}$ $$\int_0^1 \frac{1}{\sqrt{t}}\,dt = \lim_{x\to 0} \int_x^1 \frac{1}{\sqrt{t}}\,dt$$

$$= \lim_{x\to 0^+} 2\sqrt{t}\,\Big|_x^1 = \lim_{x\to 0^+} 2\left[\sqrt{1}-\sqrt{x}\right] = \lim_{x\to 0^+} 2\left(1-\sqrt{x}\right) = 2$$

$$\int_0^1 \frac{1}{\sqrt{t}}\,dt = 2$$

$\int_0^1 \frac{1}{t^r}\,dt$ eksistere for alle $r<1$.

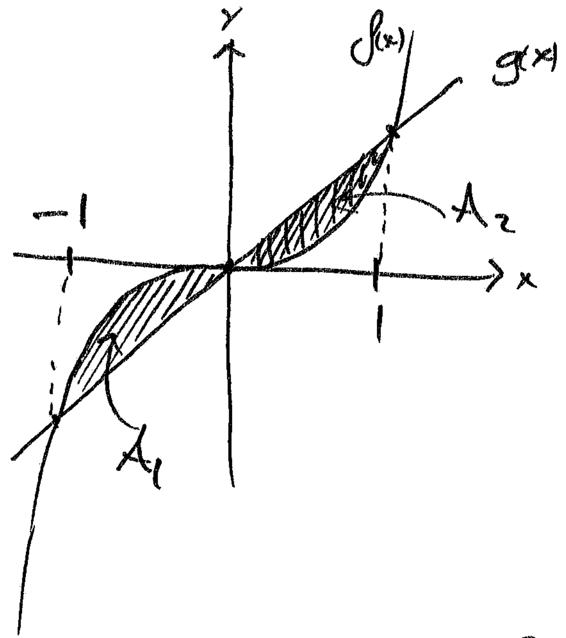
Det er lik $\int_0^1 t^{-r}\,dt = \lim_{x\to 0^+} \int_x^1 t^{-r}\,dt$

$$= \lim_{x\to 0^+} \frac{t^{-r+1}}{1-r}\Big|_x^1 = \lim_{x\to 0^+} \frac{1}{1-r}\left(1-x^{1-r}\right)$$

$$= \underline{\underline{\frac{1}{1-r}}}$$

Eksempel  Bestem arealet avgrenset av grafen til $f(x) = x^3$ og $g(x) = x$

⑤




$$f(x) = g(x)$$

$$x^3 = x$$

$$x(x^2 - 1) = 0$$

grafen til $f$ og $g$ treffer hverandre når $x = 0$, $x = -1$ og $x = 1$

$$A_2 = \int_0^1 (g(x) - f(x))\,dx = \int_0^1 g(x)\,dx - \int_0^1 f(x)\,dx$$

$$A_1 = \int_{-1}^0 (f(x) - g(x))\,dx$$

$g(x)$ og $f(x)$ er odde funksjoner, så grafen er symmetrisk om origo. Derfor er $A_1 \doteq A_2$

$$A_2 = \int_0^1 x - x^3\,dx = \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{4} - 0 = \underline{\underline{\frac{1}{4}}}$$

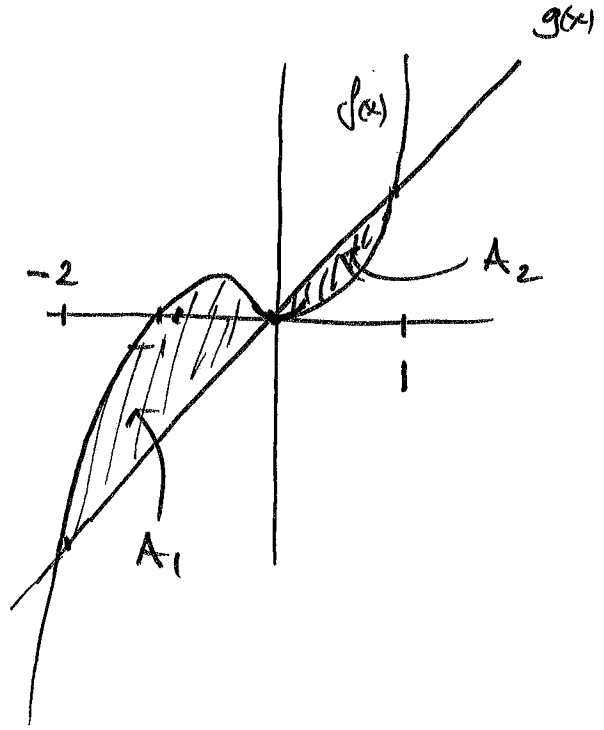
Så arealet avgrenset av $f$ og $g$ er

$$A_1 + A_2 = \frac{1}{4} + \frac{1}{4} = \underline{\underline{\frac{1}{2}}}$$

⑥ Finn arealet avgrenset av grafen til

$f(x) = x^3 + x^2$ og

$g(x) = 2x$ .




$f(x) = g(x)$

$x^3 + x^2 = 2x$

$x(x^2 + x - 2) = 0$

$x(x+2)(x-1) = 0$

$$A_1 = \int_{-2}^{0} f(x) - g(x)\, dx$$

$$A_2 = \int_{0}^{1} g(x) - f(x)\, dx$$

$$\int f(x) - g(x)\, dx = \int x^3 + x^2 - 2x\, dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} - x^2 + C$$

$$A_1 = \frac{x^4}{4} + \frac{x^3}{3} - x^2 \Big|_{-2}^{0} = -\left[\frac{(-2)^4}{4} + \frac{(-2)^3}{3} - (-2)^2\right]$$

$$= -\left[4 - \frac{8}{3} - 4\right] = \underline{\frac{8}{3}}$$

$$A_2 = -\left[\frac{x^4}{4} + \frac{x^3}{3} - x^2\right]\Big|_{0}^{1} = -\left[\frac{1}{4} + \frac{1}{3} - 1\right]$$

$$= 1 - \frac{1}{4} - \frac{1}{3} = 1 - \frac{7}{12} = \underline{\underline{\frac{5}{12}}}$$

Arealet er $\frac{5}{12} + \frac{8}{3} = \frac{5}{12} + \frac{32}{12} = \underline{\underline{\frac{37}{12}}}$