EKSEMPLER

28 feb. 2012

① Løs likningen: $2\log_2 x = 3\log_3 x + 1$

oppg (Husk at $\log_a x = \dfrac{\log x}{\log(a)}$, $a^{\log_a x} = x$)

Likningen er ekvivalent til $2 \cdot \dfrac{\log x}{\log 2} = 3 \cdot \dfrac{\log x}{\log 3} + 1$

Linær likning i $\log x$:

$$\left(\frac{2}{\log 2} - \frac{3}{\log 3}\right)\log x = 1.$$

$$\log x = 2{,}80783$$

$$x = 10^{2{,}80783..} \sim \underline{642},$$

eks Løs likningen:

$$\log(x+1) - \log x = \log(x+3)$$

$$\Leftrightarrow \quad \log(x+1) = \log(x+3) + \log x$$

$$= \log((x+3)\cdot x)$$

$$\Leftrightarrow \quad x+1 = (x+3)\cdot x$$

$$= x^2 + 3x$$

(og $x > 0$) siden $\log x$ bare er definert for $x > 0$

$$\Leftrightarrow \quad x^2 + 2x - 1 = 0$$

$$(x+1)^2 - 1 - 1 = 0$$

$$(x+1)^2 = 2$$

$x + 1 = \pm\sqrt{2}$, $\underline{\underline{x = -1 + \sqrt{2}}}$ ikke mulig $\cancel{x = -1 - \sqrt{2}}$

Eksempel ② Løs likningen $3^{x+1} = 5 \cdot 2^x$

$$\Leftrightarrow \quad \log(3^{x+1}) = \log(5 \cdot 2^x)$$

$$(x+1) \cdot \log 3 = \log(5) + \log 2^x$$

$$= \log(5) + x \cdot \log 2$$

Lineær likning i x. Løser for x:

$$x \cdot \log 3 + \log 3 = \log 5 + x \cdot \log 2$$

$$x(\log 3 - \log 2) = \log 5 - \log 3$$

$$\underline{x = \frac{\log 5 - \log 3}{\log 3 - \log 2}} = 1.2598\ldots$$

oppg. Løs likningen $4^x - 2^{x+1} - 15 = 0$

(Hint 1: $4^x = (2^2)^x = 2^{2x} = (2^x)^2$)

(Hint 2: $2^{x+1} = 2^x \cdot 2^1 = 2 \cdot 2^x$)

$$(2^x)^2 - 2 \cdot 2^x - 15 = 0 \qquad 2^x = v$$

$$v^2 - 2v - 15 = 0$$

$$(v-5)(v+3) = 0$$

Løsningen: $v = -3$, $v = 5$

$2^x = -3$ ingen løsning

$$2^x = 5$$

$$\log 2^x = x \cdot \log 2 = \log 5$$

$$x = \frac{\log 5}{\log 2} = \underline{\underline{2{,}3219\ldots}}$$

③ Følgende er lik rasjonale tall.

1) $\text{Log } \sqrt[3]{10^7}$, 2) $\text{Log } 2 + \text{Log } 50$

3) $\dfrac{\text{Log } 256}{\text{Log } 32}$

Finn de rasjonale tallene.

1) $\text{Log } \sqrt[3]{10^7} = \text{Log}\left(\left(10^7\right)^{1/3}\right) = \text{Log}\left(10^{7/3}\right)$

$= \underline{\underline{7/3}}$

2) $\text{Log } 2 + \text{Log } 50 = \text{Log } (2 \cdot 50)$

$= \text{Log } (100) = \text{Log } (10^2) = \underline{\underline{2}}$

3) $32 = 2^5$ $256 = 2^8$

$$\frac{\text{Log } 256}{\text{Log } 32} = \frac{\text{Log } 2^8}{\text{Log } 2^5}$$

$$= \frac{8 \cdot \text{Log } 2}{5 \cdot \text{Log } 2}$$

$$= \underline{\underline{\frac{8}{5}}}$$

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11.4 og 11.8 Euler-tallet

og den deriverte til eksponentialfunksjoner.

$a > 0 \qquad a \neq 1$

Hva er $\frac{d}{dx} a^x$?

Fra definisjonen er

$$\frac{d}{dx} a^x = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

merk at $a^{x+h} = a^x \cdot a^h$. Derfor er

$$\frac{d}{dx} a^x = \lim_{h \to 0} a^x \cdot \frac{a^h - 1}{h}$$

Resultat: $\lim_{h \to 0} \frac{a^h - 1}{h}$ eksisterer.

$$\frac{d}{dx} a^x = a^x \cdot \left(\lim_{h \to 0} \frac{a^h - 1}{h} \right)$$

Når er konstanten $\lim_{h \to 0} \frac{a^h - 1}{h}$ lik 1?

Det skjer når a er Eulertallet

$$e = 2,718281828459\ldots$$

La $h = \frac{1}{n}$
$h \to 0$ når $n \to \infty$ naturlige tall

$$\lim_{n \to \infty} \frac{a^{1/n} - 1}{1/n} = \lim_{n \to \infty} \frac{1/n}{1/n}$$

$$\Leftrightarrow \lim_{n \to \infty} \frac{a^{1/n} - (1 + 1/n)}{1/n} = 0$$

$$\Leftrightarrow \lim_{n \to \infty} n\left(a^{1/n} - \left(1 + \frac{1}{n}\right)\right) = 0$$

Dette indikerer at $a = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$.

⑤ Euler tallet $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

Til orientering: $e = 1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\cdots+\frac{1}{n!}+\cdots$

$n! = 1\cdot 2\cdot 3\cdots n.$

$$e^x = 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\cdots+\frac{x^n}{n!}+\cdots$$

$$\boxed{\frac{d}{dx}e^x = e^x}$$

$u = 2x-1$

eksempler

$$\left(e^{2x-1}\right)' = \frac{de^u}{du}\cdot\frac{du}{dx} \quad \text{kjerneregelen}$$

$$= e^u\cdot(2x-1)'$$

$$= \underline{2e^{2x-1}}$$

—

$$\left(x\cdot e^x - e^x\right)'$$

$$= (xe^x)' - (e^x)' \quad \text{produktregelen}$$

$$= (x)'\cdot e^x + x(e^x)' - e^x$$

$$= 1\cdot e^x + xe^x - e^x$$

$$= \underline{xe^x}$$

—

$$\left(e^{-x^2}\right)' \underset{\text{kjerneregelen}}{=} e^{-x^2}(-x^2)' = \underline{-2x\,e^{-x^2}}$$

(6)

$\text{Log}_e = \ln$ naturlig logaritme.

$$\ln x = \frac{\text{Log}\, x}{\text{Log}\, e} \qquad e^{\ln x} = x \qquad x > 0$$

$$\text{Log}\, x = \frac{\ln x}{\ln 10} \qquad \ln 10 = 2{,}3025\ldots$$

$$10^x = \left(e^{\ln 10}\right)^x = e^{(\ln 10)\cdot x}$$

$(\ln 10)\, x = U$

$U' = \ln 10$

$$\frac{d}{dx} 10^x = \frac{d}{dx} e^{(\ln 10)\cdot x}$$

$$= \frac{d\,e^U}{dU} \cdot \frac{dU}{dx}$$

$$= e^U \cdot \ln 10$$

$$\frac{d}{dx} 10^x = \ln 10 \cdot 10^x$$

$$= (2{,}3025\ldots) \cdot 10^x$$

$$a^x = \left(e^{\ln a}\right)^x = e^{\ln a \cdot x}$$

Dette gir som ovenfor

$$\underline{\frac{d}{dx} a^x = \ln(a) \cdot a^x}$$

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$$e^{\ln x} = x \qquad x > 0$$

De deriverte er også like for $x > 0$:

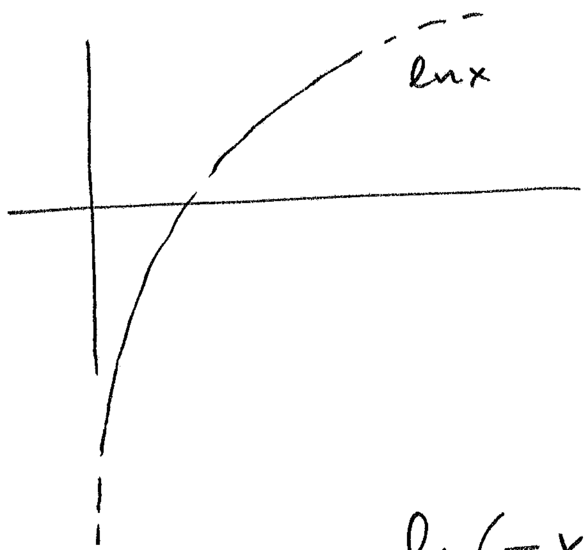
$$\frac{d}{dx}\left(e^{\ln x}\right) = \frac{d\,e^{u}}{du} \cdot \frac{du}{dx} \qquad u = \ln x$$

$$= e^{\ln x} \cdot \frac{d \ln x}{dx}$$

er derfor lik $\frac{d}{dx} x = 1$

Så $$\frac{d \ln x}{dx} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\boxed{\frac{d \ln x}{dx} = \frac{1}{x}} \qquad x > 0$$



$\ln(-x)$ definert for $x < 0$

$$\frac{d}{dx} \ln(-x) = \frac{d \ln u}{du} \cdot \frac{du}{dx} \qquad u = -x, \quad u' = -1$$

$$= \frac{1}{u} \cdot (-1)$$

$$= \frac{1}{-x} \cdot (-1) = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \ln|x| = \frac{1}{x} \qquad x \neq 0}$$