12 jan 2012 Repetisjon om potenser

(1)

$$x^n = \overbrace{x \cdots x}^{n} \qquad n \text{ naturlig tall.}$$

$$x^r \cdot x^s = x^{r+s} \qquad x^1 = x$$

$$(x^r)^s = x^{r \cdot s} \qquad x^0 = 1$$

$$x^{-1} = \frac{1}{x} \quad (x \neq 0)$$

$$x^{-n} = (x^{-1})^n = \left(\frac{1}{x}\right)^n = \frac{1}{x^n} \qquad n \text{ naturlig tall}$$

$$x^{1/n} = \sqrt[n]{x} \quad \text{def for} \quad \begin{array}{ll} x \geq 0 & n \text{ jevn nat. tall} \\ \text{alle } x & n \text{ odde nat. tall.} \end{array}$$

$$x^{m/n} = (x^{1/n})^m = (\sqrt[n]{x})^m$$

$$= (x^m)^{1/n} = \sqrt[n]{x^m}. \qquad \begin{array}{ll} x \geq 0 & \frac{m}{n} \text{ positiv} \\ x > 0 & \text{ellers} \end{array}$$

Merk: $$x^{2/2} = x^1 = x \qquad x \geq 0$$

$$= \sqrt{x^2} = |x|$$

$$x^r = \lim_{\frac{m}{n} \to r} x^{m/n}$$

$$x^\pi = \lim_{\frac{m}{n} \to \pi} x^{\frac{m}{n}} \qquad \pi = 3.14159\ldots$$

x^π er grensen til følgen

$$\{x^3, x^{3.1}, x^{3.14}, x^{3.141}, x^{3.1415}, \ldots\}$$

②   9.1   Derivasjonsregler

Resultat $\frac{d}{dx} x^r = r x^{r-1}$ (når x^{r-1} er definert)

eksempel $(\sqrt[3]{x})' = (x^{1/3})' = \frac{1}{3} \cdot x^{\frac{1}{3}-1}$

$= \frac{1}{3} \cdot x^{-2/3} = \underline{\frac{1}{3\sqrt[3]{x^2}}}$

$\left(\frac{1}{\sqrt[3]{x}}\right)' = \left(\left(\sqrt[3]{x}\right)^{-1}\right)' = \left(\left(x^{1/3}\right)^{-1}\right)' = \left(x^{-1/3}\right)'$

$= \frac{-1}{3} \cdot x^{-\frac{1}{3}-1} = \frac{-1}{3} x^{-4/3} = \underline{\frac{-1}{3\sqrt[3]{x^4}}}$

$\left(\sqrt[4]{x^5}\right)' = \left(\left(x^5\right)^{1/4}\right)' = \left(x^{5/4}\right)'$

$= \frac{5}{4} x^{5/4-1} = \frac{5}{4} x^{1/4} = \underline{\frac{5}{4} \cdot \sqrt[4]{x}}$

$\left(3\sqrt{x} + 2x^3\right)' = 3(\sqrt{x})' + 2(x^3)'$

$= 3(x^{1/2})' + 2(x^3)'$

$= 3 \cdot \frac{1}{2} \cdot x^{-1/2} + 2 \cdot 3 \cdot x^2$

$= \underline{\frac{3}{2\sqrt{x}} + 6 \cdot x^2}.$

④

$$\frac{d}{dx} f(ax+b) = a \cdot f'(ax+b)$$

$$u(x) = ax+b \qquad u'(x) = a$$

$$\frac{d}{dx} f(u(x)) = a \cdot \frac{d}{du} f(u)$$

eks. $f(x) = (x+1)^2 = x^2 + 2x + 1$

$$f'(x) = (x^2+2x+1) = 2x+2 = 2(x+1)$$

$$g(u) = u^2 \qquad u = (x+1). \qquad g'(u) = 2 \cdot u$$

$$f(x) = g(u(x)) = g(x+1).$$

$$\frac{d}{dx} f(x) = 1 \cdot g'(x+1) = 1 \cdot 2(x+1) = 2(x+1)$$

$$\left(\left(3x-4\right)^{100}\right)' \qquad g(u) = u^{100}, \; g'(u) = 100 \cdot u^{99}$$

$$u = 3x-4, \; u' = 3$$

$$= \underline{3 \cdot 100 (3x-4)^{99}}$$

$$f(x) = (x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$f'(x) = (x^3+3x^2+3x+1)' = 3x^2 + 6x + 3$$

$$= 3(x^2+2x+1) = 3(x+1)^2$$

$$g(u) = u^3 \qquad u = x+1$$

$$g'(u) = 3u^2 \qquad u' = 1$$

$$f'(x) = 1 \cdot 3(x+1)^2 = \underline{3(x+1)^2}$$

③ opg.

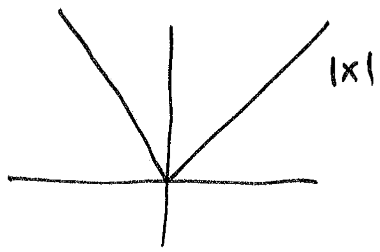
$$(x\cdot\sqrt{x})' = (x^{1}\cdot x^{1/2})' = (x^{1+\frac{1}{2}})'$$
$$= (x^{3/2})' = \frac{3}{2}\cdot x^{3/2-1} = \frac{3}{2}\cdot x^{1/2}$$
$$= \underline{\frac{3}{2}\cdot\sqrt{x}}$$

$$\left(\frac{1}{\sqrt[5]{x}}\right)' = \left(\frac{1}{x^{1/5}}\right)' = \left(\left(x^{1/5}\right)^{-1}\right)' = \left(x^{-1/5}\right)'$$
$$= \frac{-1}{5}\cdot x^{-\frac{1}{5}-1} = \frac{-1}{5}\cdot x^{-6/5}$$
$$= \underline{\frac{-1}{5\sqrt[5]{x^{6}}}}$$

$$\left(\sqrt[3]{\sqrt{x}}\right)' = \left(\left(x^{1/2}\right)^{1/3}\right)' = \left(x^{\frac{1}{2}\cdot\frac{1}{3}}\right)'$$
$$= \left(x^{1/6}\right)' = \frac{1}{6}\cdot x^{\frac{1}{6}-1}$$
$$= \frac{1}{6}\cdot x^{-5/6} = \underline{\frac{1}{6\sqrt[6]{x^{5}}}}$$

$$\left(\sqrt{x^{2}}\right)' = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$ eksisterer ikke når $x=0$.

$$\sqrt{x^{2}} = |x| = \begin{cases} x & x\geq 0 \\ -x & x\leq 0 \end{cases}$$



$$\left(\frac{\sqrt{x}}{\sqrt[3]{x}}\right)' = \left(\frac{x^{1/2}}{x^{1/3}}\right)' = \left(x^{1/2}\left(x^{1/3}\right)^{-1}\right)'$$
$$= \left(x^{1/2}\cdot x^{-1/3}\right)' = \left(x^{\frac{1}{2}-\frac{1}{3}}\right)' = \left(x^{1/6}\right)'$$
$$= \underline{\frac{1}{6}\,x^{-5/6}}$$

$$(x^{\sqrt{2}})' = \underline{\sqrt{2}\cdot x^{\sqrt{2}-1}}\,, \qquad (x^{\pi})' = \pi\cdot x^{\pi-1}.$$
$$(x^{1.72})' = \underline{(1.72)\,x^{0.72}}$$

5

Forslag

opg $\left(\sqrt{4x-3}\right)' = \frac{1}{2}\frac{1}{\sqrt{4x-3}}$?

$\frac{1}{\sqrt{x}}$?

$\frac{1}{\sqrt{8x-2}}$?

$2x^{-1/2}$?

$x^{-1/2}$?

$u = 4x-3$

$g(u) = \sqrt{u}$ $g'(u) = (u^{1/2})' = \frac{1}{2\sqrt{u}}$

$\left(\sqrt{4x-3}\right)' = \left(g(4x-3)\right)' = 4 \cdot g'(4x-3)$

$= 4 \cdot \frac{1}{2\sqrt{4x-3}} = \underline{\frac{2}{\sqrt{4x-3}}}$

Bevis for resultatet $\frac{d}{dx} f(ax+b) = a f'(ax+b)$.

Når $a = 0$: $\frac{d}{dx} f(b) = 0$ og $0 \cdot f'(0 \cdot x + b) = 0$. ok

Anta $a \neq 0$. $\frac{d}{dx} f(ax+b) = \lim_{h \to 0} \frac{f(a(x+h)+b) - f(ax+b)}{h}$

$= a \cdot \lim_{h \to 0} \frac{f(ax+b+a \cdot h) - f(ax+b)}{a \cdot h}$ La $a \cdot h = k$

siden $a \neq 0$ er dette lik

$a \lim_{k \to 0} \frac{f(ax+b+k) - f(ax+b)}{k} = a \cdot f'(ax+b)$.