13 feb 2009

10.7 Trigonometriske ulikheter

1) $\sin x > 1/2$

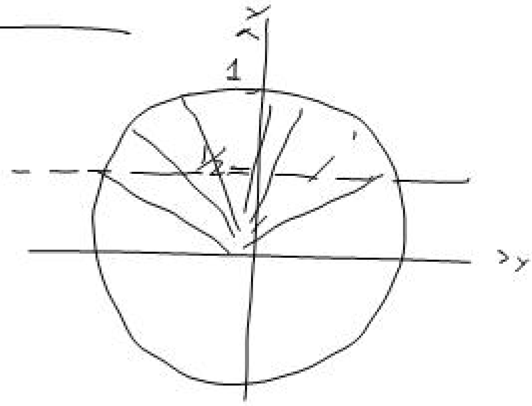
2) $-\cos x \leq \frac{\sqrt{3}}{2}$ $\qquad x \in [0, 4\pi]$

3) $2\sin\left(\frac{\pi}{4} - 2\pi x\right) \leq \sqrt{2}$

4) $\sin x > \cos x - 1$

5) $\tan x > \sqrt{3}$ $\qquad x \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

Løsninger



1) $\sin x > \frac{1}{2}$

$\sin x = \frac{1}{2}$ for

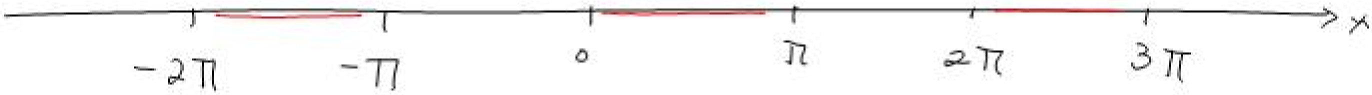
$x = \frac{\pi}{6} + 2\pi \cdot n$

og $x = \pi - \frac{\pi}{6} + 2\pi \cdot n = \frac{5\pi}{6} + 2\pi \cdot n$.

Løsningen til ulikheten $\sin x > 1/2$

er $\frac{\pi}{6} + 2\pi n < x < \frac{5\pi}{6} + 2\pi \cdot n$ $\qquad$ n heltall

Rødt område: løsningsmengde

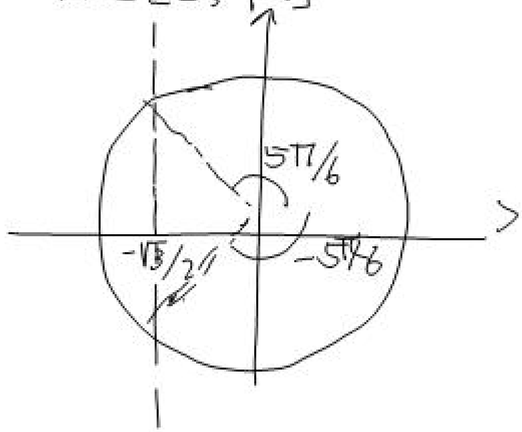


2) $-\cos x \leq \sqrt{3}/2$

$-1\,(-\cos x \leq \sqrt{3}/2)$

$\cos x \geq -\sqrt{3}/2$

$x \in [0, 4\pi]$



$\cos x = -\frac{\sqrt{3}}{2}$ $\quad x = \frac{5\pi}{6} + 2\pi \cdot n$

og $x = \frac{-5\pi}{6} + 2\pi \cdot n$

$= \frac{7\pi}{6} + 2\pi(n-1)$

Løsningen til $\cos x \geq -\sqrt{3}/2$ er

$\frac{-5\pi}{6} + 2\pi \cdot n \leq x \leq \frac{5\pi}{6} + 2\pi \cdot n$ $\quad$ n heltall.

Løsningsmengden til $\cos x \geq -\frac{\sqrt{3}}{2}$ $\quad x \in [0, 4\pi]$

er $\underline{\left[0, \frac{5\pi}{6}\right] \cup \left[\frac{7\pi}{6}, \frac{5\pi}{6} + 2\pi\right] \cup \left[\frac{7\pi}{6} + 2\pi, 4\pi\right]}$

3) $$2 \sin\left(\frac{\pi}{4} - 2\pi x\right) \le \sqrt{2}$$

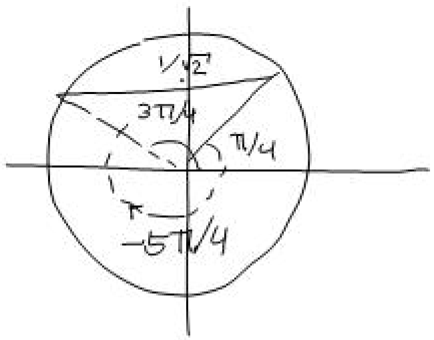
Deler på $2 (>0)$:

$$\sin\left(\frac{\pi}{4} - 2\pi x\right) \le \frac{1}{\sqrt{2}}$$

(Merk at $\frac{\sqrt{2}}{2} = \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}} \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$.)

La $u = \frac{\pi}{4} - 2\pi x$.

Vi løser ulikheten $\sin u \le \frac{1}{\sqrt{2}}$.



Løsningene til $\sin u = \frac{1}{\sqrt{2}}$ er

$$u = \frac{\pi}{4} + 2\pi \cdot n$$

$$u = \frac{3\pi}{4} + 2\pi \cdot n$$

n heltall.

Løsningene til $\sin u \le \frac{1}{\sqrt{2}}$

$$-\frac{5\pi}{4} + 2\pi \cdot n \le u \le \frac{\pi}{4} + 2\pi \cdot n$$, n heltall.

Vi løser nå for x :

Setter inn $u = \frac{\pi}{4} - 2\pi \cdot x$

$$-\frac{5\pi}{4} + 2\pi \cdot n \le \frac{\pi}{4} - 2\pi \cdot x \le \frac{\pi}{4} + 2\pi \cdot n$$

trekker ifra $\frac{\pi}{4}$ og deler med π

$$-\frac{6}{4} + 2n \le -2x \le 2n$$

deler med -2 (<0)

$\longrightarrow$

$$+\frac{3}{4} - n \geq x \geq -n$$

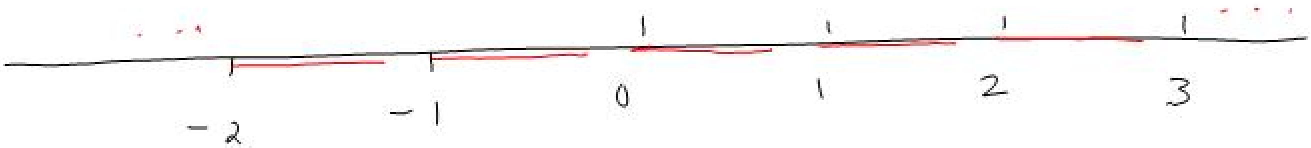
Vi har funnet at løsningene til den trigonometriske ulikheten

$$2\sin\left(\frac{\pi}{4} - 2\pi x\right) \leq \sqrt{2}$$

er $\quad m \leq x \leq \frac{3}{4} + m$

for heltall m

(Vi har byttet $-n$ med m for ordens skyld.)



4) $$\sin x > \cos x - 1$$

$$\sin x - \cos x > -1$$

Sin x og cos x er sinusbølger med samme frekvens, Derfor er sin x − cos x en sinusbølge (med samme frekvens).

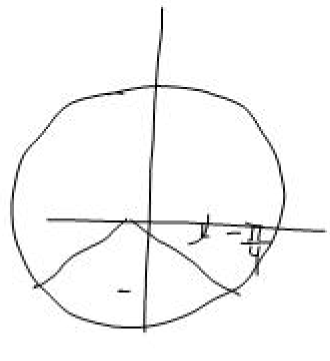
$$\sin x - \cos x = +\sqrt{2}\,\sin\left(x - \frac{\pi}{4}\right)$$

OPPGAVE: Syn dette! (se på notatene fra mandag 9 februar.)

$$\sqrt{2}\sin\left(x-\frac{\pi}{4}\right) > -1$$

$$\sin\left(x-\frac{\pi}{4}\right) > \frac{-1}{\sqrt{2}}$$

La $\quad u = x - \frac{\pi}{4}$



$$\sin(u) > \frac{-1}{\sqrt{2}}$$

Løsningen er

$$-\frac{\pi}{4} + 2\pi\cdot n < u < \frac{5\pi}{4} + 2\pi\cdot n$$

n heltall.

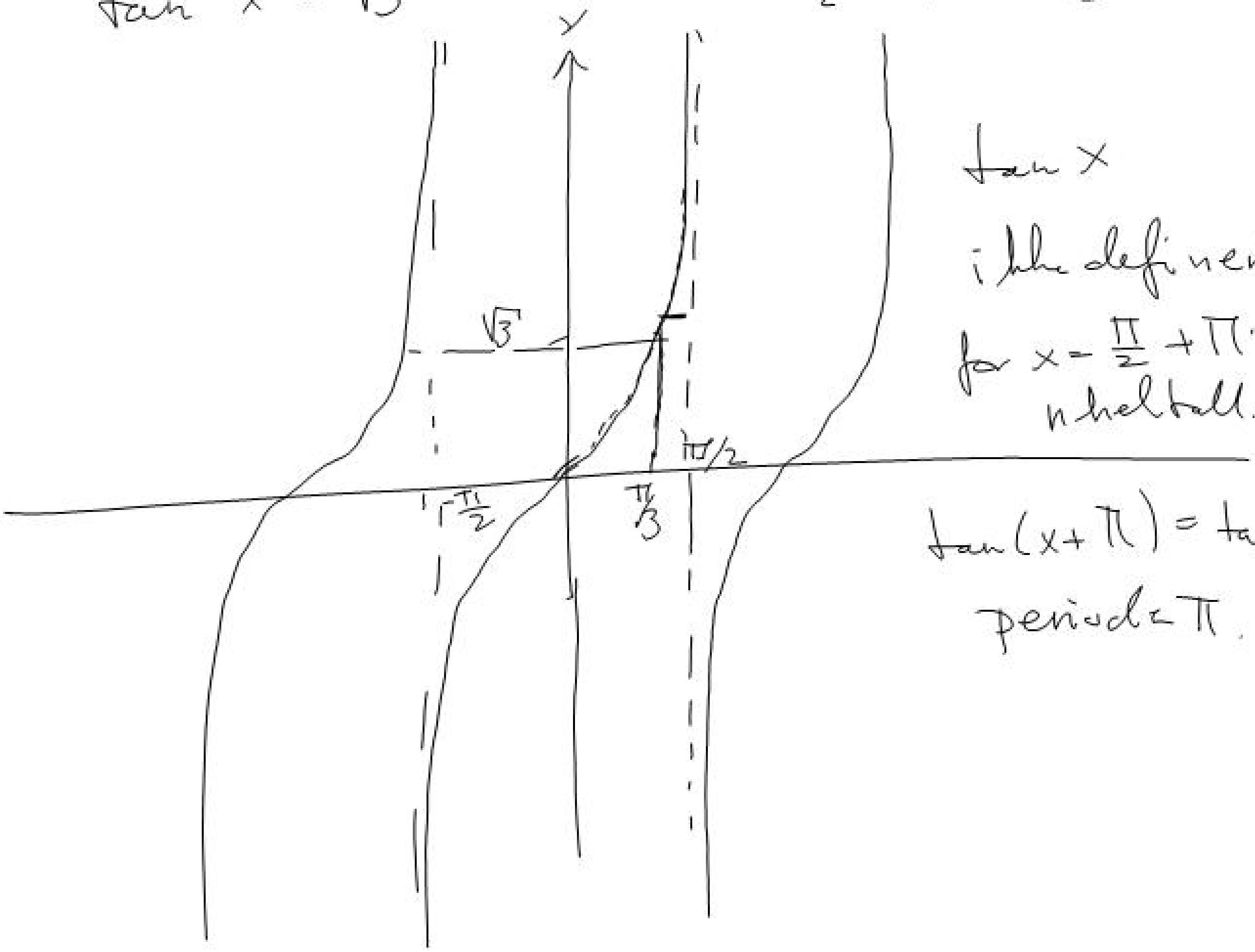
$$-\frac{\pi}{4} + 2\pi\cdot n < \overbrace{x-\frac{\pi}{4}}^{u} < \frac{5\pi}{4} + 2\pi\cdot n$$

(legger til $\frac{\pi}{4}$)

$$2\pi\cdot n < x < \frac{6\pi}{4} + 2\pi\cdot n$$

$$\underline{2\pi\cdot n < x < \frac{3\pi}{2} + 2\pi\cdot n}$$

$\tan x > \sqrt{3}$ $\quad -\frac{\pi}{2} < x < \frac{\pi}{2}$



$\tan x$

ikke definert

for $x = \frac{\pi}{2} + \pi \cdot n$

n heltall.

$\tan(x+\pi) = \tan x$

periode π.

$\tan x = \sqrt{3}$ løsninger: $\frac{\pi}{3} + \pi \cdot n$

Fra grafen ser vi at løsningen til

$\tan x > \sqrt{3}$ for $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

er $\underline{\frac{\pi}{3} < x < \frac{\pi}{2}}$