LØSNINGSFORSLAG

Oppg. 1 a) $y'' + y = t$

Løser først $y'' + y = 0$

Kar. lign. $r^2 + 1 = 0 \iff r = \pm i$

Løsning $e^{it} = \cos t + i \sin t$

Gen. løsn. $y_h = c_1 \cos t + c_2 \sin t$

Søker part løsn. $y_p = at + b$

$y_p' = a$, $y_p'' = 0$

innsatt : $at + b = t \iff a = 1, b = 0$

Gen. løsn. av $y'' + y = t$:

$$y = y_h + y_p = \underline{\underline{c_1 \cos t + c_2 \sin t + t}}$$

b)

$$y = t \cos t$$
$$y' = \cos t + t(-\sin t)$$
$$y'' = -\sin t - \sin t + t(-\cos t) = -2\sin t - t\cos t$$

Innsatt i $y'' + y = -2\sin t$

$$-2\sin t - t\cos t + t\cos t = -2\sin t \quad \text{OK}$$

Gen løsning

$$y = y_h + y_p = c_1 \cos t + c_2 \sin t + t \cos t$$
$$y(0) = c_1 = 0$$
$$y' = c_2 \cos t + t(-\sin t) + \cos t$$
$$y'(0) = c_2 + 1 = 4 \iff c_2 = 3$$

Løsn. av initialverdiproblem :

$$y = \underline{\underline{3\sin t + t\cos t}}$$

Når $t \to \infty$, så vil "amplituden" i $t\cos t$ gå mot $\pm\infty$ fordi når $\cos t = \pm 1$, så vil $t\cos t = \pm t$. Vi får <u>RESONANS</u>

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Oppg. 2 $\quad z = x^3 + 2y^3 - 3x - 3y^2 + 1$

a) $z_x = \underline{\underline{3x^2 - 3}}$, $z_y = \underline{\underline{6y^2 - 6y}}$

$z_{xx} = \underline{\underline{6x}}$, $z_{yy} = \underline{\underline{12y - 6}}$, $z_{xy} = z_{yx} = \underline{\underline{0}}$

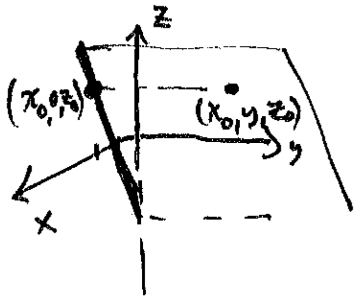
b) Da $f(2,1) = 8 + 2 - 3 \cdot 2 - 3 \cdot 1 + 1 = 2$, så ligger $(2,1,2)$ på flaten.

Tangentplanets ligm.:

$$z - 2 = z_x(2,1)(x-2) + z_y(2,1)(y-1)$$

$$z - 2 = 9(x-2) + 0(y-1)$$

$$z = 9x - 18 + 2 \iff z = \underline{\underline{9x - 16}}$$



Planet er parallell med $\underline{\underline{y\text{-aksen}}}$ fordi hvis $(x_0, 0, z_0)$ passer i ligm., så passer (x_0, y, z_0) for alle valg av y.

c) $c = f(2,2) = 2^3 + 2 \cdot 2^3 - 3 \cdot 2 - 3 \cdot 2^2 + 1 = 8 + 16 - 6 - 12 + 1 = 7$

Det totale differensialet, tatt med tallverdi på hvert ledd, gir oss max. usikkerhet i c, dvs

$$|z_x(2.2)\,dx| + |z_y(2,2)\,dy| \quad \text{der} \quad \begin{array}{l} dx = 0.02 \\ dy = 0.03 \end{array}$$

Får

$$(3 \cdot 2^2 - 3)\,dx + (6 \cdot 2^2 - 6 \cdot 2)\,dy = 9\,dx + 12\,dy =$$

$$9 \cdot 0.02 + 12 \cdot 0.03 = 0.18 + 0.36 = \underline{\underline{0.54}}$$

gir oss max usikkerhet i c, dvs

$$\underline{\underline{c = 7.00 \pm 0.54}}$$

d) Kritiske punkter

$z_x = 3(x^2 - 1) = 0$, $z_y = 6y(y-1) = 0$

$\iff x = \pm 1$, $y = 0, y = 1$

4 kritiske punkter :

$\underline{\underline{(1,0) , (-1,0) , (1,1) , (-1,1)}}$

3

$$\Delta = z_{xx}\cdot z_{yy} - z_{xy}^2 = 6x\cdot 6(2y-1) - 0^2 = \underline{36x(2y-1)}$$

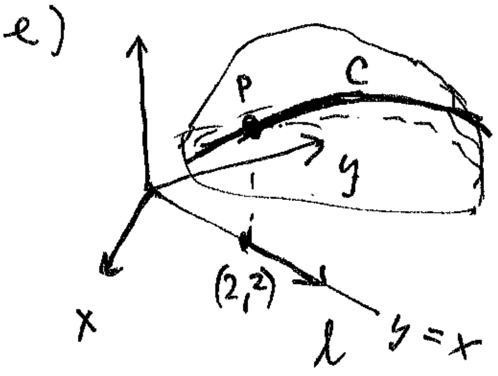
$\Delta(1,0) = 36(-1) = -36 < 0 \implies$ $\underline{\underline{(1,0) \text{ er sadelpunkt}}}$

$\Delta(-1,0) = 36(-1)(-1) = 36 > 0$
og $z_{xx}(-1,0) = -6 < 0 \implies$ $\underline{\underline{(-1,0) \text{ er max.punkt}}}$

$\Delta(1,1) = 36\cdot(2-1) = 36 > 0$, $z_{xx} = 6 > 0 \implies$ $\underline{\underline{(1,1) \text{ er min. punkt}}}$

$\Delta(-1,1) = 36(-1)(2-1) = -36 < 0 \implies$ $\underline{\underline{(-1,1) \text{ er sadelpunkt}}}$

e)



En retningsvektor for l er
$\vec{v} = [1,1]$.

Stign. tallet i $P = (2,2,7)$ til tangenten er den retnings-derivert i $(2,2)$ i retn. $\vec{v}$, dvs

$$D_f(2,2) = \nabla f(2,2)\cdot\frac{\vec{v}}{|\vec{v}|} \underset{\uparrow}{=} [9,12]\cdot\frac{[1,1]}{\sqrt{2}} = \frac{21}{\sqrt{2}} = \underline{\underline{\frac{21\sqrt{2}}{2}}}$$

se c)

Opp. 3 a) $\displaystyle\sum_{n=0}^{\infty}\frac{2^{n+2}}{7^n}$, $a_n = \dfrac{2^{n+2}}{7^n} \implies \dfrac{a_{n+1}}{a_n} = \dfrac{2^{n+3}}{7^{n+1}}\cdot\dfrac{7^n}{2^{n+2}} = \dfrac{2}{7}$

$\dfrac{a_{n+1}}{a_n}$ er uavhengig av n. Rekka er geom og $k = \frac{2}{7}$ oppfyller $|k| < 1$. Rekka konv. og summen er

$$s = \frac{a_0}{1-k} = \frac{4}{1-\frac{2}{7}} = \frac{4\cdot 7}{7-2} = \underline{\underline{\frac{28}{5}}}$$

$\displaystyle\sum\frac{n-1}{n^{3/2}+2n^2}$. La $a_n = \dfrac{n-1}{n^{3/2}+2n^2}$ og $b_n = \dfrac{n}{2n^2} = \dfrac{1}{2n}$

Rekka $\sum b_n = \frac{1}{2}\cdot\sum\frac{1}{n}$ (p-rekke med $p=1$) divergerer. Derfor divergerer rekka $\sum a_n$.

b) R kalles <u>konvergensradius</u>.

Gitt $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{n \cdot 2^n}$. La $a_n = (-1)^n \frac{x^{2n}}{n \cdot 2^n}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{x^{2n+2}}{(n+1)\, 2^{n+1}} \cdot \frac{n \cdot 2^n}{x^{2n}} \right| = \frac{n}{n+1} \cdot \frac{x^2}{2} \underset{(n \to \infty)}{\longrightarrow} \frac{x^2}{2}$$

Rekka konv når $\frac{x^2}{2} < 1$, dvs $x^2 < 2 \iff |x| < \sqrt{2}$

og den divergerer når $|x| > \sqrt{2}$. $\Longrightarrow$

$$\underline{\underline{R = \sqrt{2}}}$$

<u>Oppg. 4</u> a) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \dots$, $|x|<1$

Erstatter x med $-x$:

$$\ln(1-x) = -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} + \dots + (-1)^{n-1} \frac{(-x)^n}{n} + \dots$$

$$= -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots - \frac{x^n}{n} - \dots = -\sum_{n=1}^{\infty} \frac{x^n}{n}, \quad |-x|<1$$

$$\Longrightarrow g(x) = \frac{\ln(1-x)}{x} = -\sum_{n=1}^{\infty} \frac{x^{n-1}}{n} = -1 - \frac{x}{2} - \frac{x^2}{3} - \dots, \quad |x|<1$$

Rekka konv. for alle <u>x med $|x|<1$</u> fordi regn. bygger på ei rekke som konv. når $|x|<1$. (Egentlig konv. rekka for $\ln(1+x)$ for $-1 < x \le 1$, dvs rekka for $g(x)$ konv. for <u>$-1 \le x < 1$</u> ved regningen over.)

b) I ei potensrekke $f(x) = \sum a_n x^n$, så er $a_n = \frac{f^{(n)}(0)}{n!}$

dvs. $f^{(n)}(0) = n! \cdot a_n$. I vårt tilfelle må vi

tolke $g^{(n)}(0) = \lim_{x \to 0} g^{n}(x)$. Vi får da

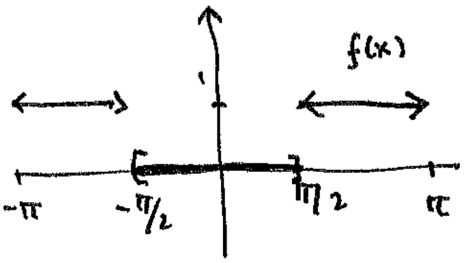
$$\lim_{x \to 0} g(x) = a_0 = \underline{\underline{-1}}, \quad \lim_{x \to 0} g^{(8)}(x) = 8! \cdot a_8 = \underline{\underline{-\frac{8!}{9}}}$$

c) Lov å integrere ledd for ledd. Får

$$\int_0^{1/2} \frac{\ln(1-x)}{x}\, dx = -\sum_{n=1}^{\infty} \frac{1}{n} \int_0^{1/2} x^{n-1}\, dx = -\sum_{n=1}^{\infty} \frac{x^n}{n^2} \Big|_0^{1/2} = \underline{\underline{-\sum_{n=1}^{\infty} \frac{1}{n^2 \cdot 2^n}}}$$

Oppg. 5

a)



Grafen er sym. om y-aksen

$\Rightarrow b_n = 0$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{\pi/2}^{\pi} 1\,dx$$

$$a_0 = \frac{1}{\pi}\cdot x \Big|_{\pi/2}^{\pi} = \frac{1}{\pi}\left(\pi - \frac{\pi}{2}\right) = \underline{\frac{1}{2}}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_{\pi/2}^{\pi}\cos nx\,dx = \frac{2}{\pi}\cdot\frac{\sin nx}{n}\Big|_{\pi/2}^{\pi}$$

$$a_n = \frac{2}{\pi n}\left(\sin n\pi - \sin\left(n\frac{\pi}{2}\right)\right) = \underline{-\frac{2}{\pi n}\sin\left(n\frac{\pi}{2}\right)}$$

$\sin\left(n\frac{\pi}{2}\right) = ?$ $\quad \sin 0 = 0, \quad \sin\frac{\pi}{2} = 1, \quad \sin\left(2\cdot\frac{\pi}{2}\right) = 0$

$\sin\left(3\cdot\frac{\pi}{2}\right) = -1, \quad \sin\left(4\frac{\pi}{2}\right) = 0 \quad$ etc

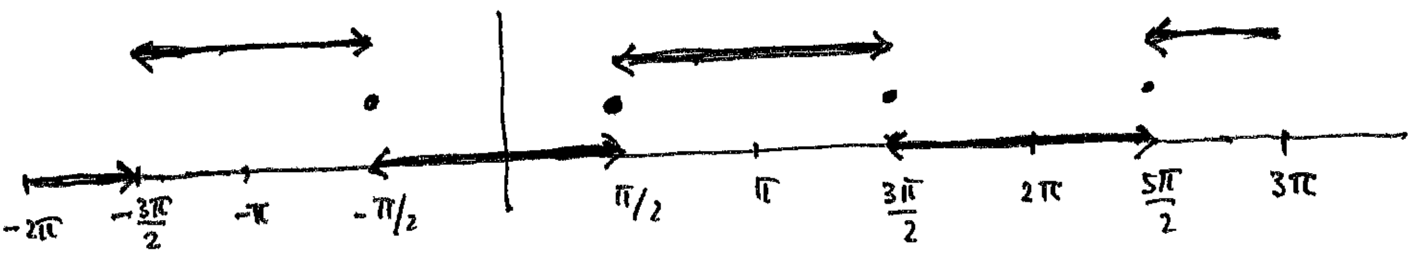
Får

$$\sin\left(n\frac{\pi}{2}\right) = \begin{cases} 0 & \text{når } n = 2k \\ (-1)^k & \text{når } n = 2k+1 \end{cases}$$

Fourierrekka er

$$\underline{\underline{f(x) \sim \frac{1}{2} - \sum_{n=1}^{\infty}\frac{2}{\pi n}\sin\left(\frac{n\pi}{2}\right)\cos nx = \frac{1}{2} - \frac{2}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{\cos(2k+1)x}{2k+1}}}$$

b) Grafen til summen = grafen til f(x) bortsett fra i diskontinuitets punktene hvor vi må ta middelverdi



Setter $x = 0$ inn i Fourierrekka:

$$0 = \frac{1}{2} - \frac{2}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{2k+1} \iff \frac{1}{2}\cdot\frac{\pi}{2} = \sum_{k=0}^{\infty}(-1)^k\frac{1}{2k+1}$$

$$\Rightarrow \sum_{k=0}^{\infty}(-1)^k\frac{1}{2k+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \underline{\underline{\frac{\pi}{4}}}$$ (Gregory's formel)